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SUPERLATIVE APPROXIMATION OF THE LUENBERGER-HICKS-MOORSTEEN PRODUCTIVITY INDICATOR: THEORY AND APPLICATION

FREDERIC ANG AND PIETER JAN KERSTENS

Abstract. Consisting of the difference between an output indicator and an input indicator, the Luenberger-Hicks-Moorsteen (LHM) productivity indicator allows straightforward interpretation. However, it requires estimation of distance functions that are inherently unknown. This paper shows that a simple Bennet profit indicator is a superlative approximation of the LHM indicator when one can assume profit-maximizing behavior and the input and output directional distance functions can be represented up to the second order by a quadratic functional form. We also show that the Luenberger- and LHM-approximating Bennet indicators coincide for an appropriate choice of the directional vectors. Focusing on a large sample of Italian food and beverages companies for the years $1995 - 2007$, we empirically investigate the extent to which this theoretical equivalence translates into similar estimates.

Keywords: Productivity and competitiveness, Bennet, Luenberger-Hicks-Moorsteen, superlative approximation, Italian food and beverages sector.

JEL classification: C43, D21, D22, D24.

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1. Introduction

Productivity analysis is an essential tool to benchmark economic performance. Following Lovell (2016), there are two approaches to productivity measurement. The *theoretical* approach to productivity measurement entails estimation of distance functions. Not requiring any estimation of distance functions, the *empirical* approach employs a simple empirical index number formula of prices and quantities of inputs and outputs. The production technology underlying distance functions is inherently unknown and their computation is often complicated, while empirical index numbers are easily computed. Therefore, finding empirical index numbers that approximate distance-function-based productivity measures is of practical interest to the empirical analyst. An important body of literature initiated by Diewert (1976) seeks to find "superlative" index numbers for which the approximation holds under the assumption of (i) economic optimizing behavior and (ii) a technology that can be represented up to the second order by a flexible functional form.

No superlative index number is currently known for the Luenberger-Hicks-Moorsteen (LHM) productivity indicator developed by Briec and Kerstens (2004). Consisting of directional output and input distance functions, the LHM indicator follows the theoretical approach. It has several attractive theoretical properties. First, it is difference-based. Ratio-based "indexes" can be undefined when zeros occur in the numerator or denominator and are not translation-invariant. Difference-based "indicators" avoid these drawbacks altogether (Balk et al., 2003). Second, the LHM indicator is additively complete, which means that it consists of the difference between an output indicator and input indicator (O'Donnell, 2012). Despite its straightforward interpretation and desirable theoretical properties, the computation of inherently unknown distance function complicates the use of the LHM indicator. The current paper addresses this issue by revealing a superlative approximation of the LHM indicator.

Our paper complements several superlative approximations from the literature. Caves et al. (1982) show that the Törnqvist index is a superlative approximation of the input (output) Malmquist index when there is cost-minimizing (revenue-maximizing) behavior, and the input and output distance functions can be represented up to the second order by a translog functional form. This result also holds for the Fisher ideal index (Balk, 1993; Färe and Grosskopf, 1992). Diewert and Fox (2010) suggest that the Törnqvist index is also a superlative approximation of Bjurek (1996)'s Hicks-Moorsteen index. Mizobuchi (2017) shows that the Törnqvist index is a superlative approximation of both the Malmquist index and Hicks-Moorsteen index under constant returns to scale. Furthermore, he demonstrates that this assumption can be loosened to α-returns-to-scale for the Hicks-Moorsteen index, but *not* for the Malmquist index. Balk (1998) and Chambers (1996, 2002) show that the Bennet profit indicator is a superlative approximation of Chambers et al. (1996)'s Luenberger indicator when there is profit-maximizing behavior, and the directional distance functions can be represented up to the second order by a quadratic functional form.

The contributions of this paper are threefold. First, we show that the Bennet profit indicator coincides with the LHM indicator when there is profit-maximizing behavior, and the input and output directional distance functions can be represented up to the second order by a quadratic functional form. The Bennet profit indicator is thus a superlative approximation of Luenberger as well as LHM indicators under equivalent theoretical conditions (albeit for a different price normalization). Second, we show the theoretical conditions under which the Luenberger-approximating Bennet indicator is equivalent to the LHM-approximating Bennet indicator. Third, focusing on a large sample of Italian food and beverages companies for the years 1995 – 2007, we empirically investigate the extent to which this theoretical equivalence translates into similar estimates. The food and beverages industry is the largest manufacturing sector in the European Union. The Italian food and beverages industry has a value added of 12.6% of the total value added of the EU-28 in 2012, which makes it the third largest contributor among the EU member states (Eurostat, 2018).

The remainder of this paper is structured as follows. The next section introduces necessary notation and definitions of Luenberger and Bennet indicators. We then define the LHM indicator and present the theoretical equivalence between the LHM and Bennet indicator. This is followed by the empirical application to the Italian food and beverages companies. The final section concludes.

2. Linking the Luenberger productivity indicator to the Bennet profit indicator

This section sets the stage for our main result by introducing necessary notation and definitions of the Luenberger indicator and the Bennet cost, revenue and profit indicators. It also reminds the reader of the result of Balk (1998) and Chambers (1996, 2002), which links the Bennet profit indicator as a superlative approximation of the Luenberger indicator.

2.1. The Luenberger productivity indicator.

Let $\mathbf{x}_t \in \mathbb{R}^n_+$ be the inputs that are used to produce outputs $\mathbf{y}_t \in \mathbb{R}^m_+$. We define the production possibility set as:

$$\boldsymbol{\mathcal{T}}_t = \left\{(\mathbf{x}_t, \mathbf{y}_t) \in \mathbb{R}^{n+m}_+ | \mathbf{x}_t \text{ can produce } \mathbf{y}_t\right\}.$$

We make the following assumptions on the production possibility set (Chambers, 2002):

Axiom 1 (Closedness). *$\boldsymbol{\mathcal{T}}_t$ is closed.*

Axiom 2 (Free disposability of inputs and outputs). *if $(\mathbf{x}'_t, -\mathbf{y}'_t) \geq (\mathbf{x}_t, -\mathbf{y}_t)$ then $(\mathbf{x}_t, \mathbf{y}_t) \in \boldsymbol{\mathcal{T}}_t \Rightarrow (\mathbf{x}'_t, \mathbf{y}'_t) \in \boldsymbol{\mathcal{T}}_t$.*

Axiom 3 (Inaction). *Inaction is possible: $(\mathbf{0}^n, \mathbf{0}^m) \in \boldsymbol{\mathcal{T}}_t$.*

The directional distance function was first introduced in a production context by Chambers et al. (1996). We denote the time-related directional distance function for $(a, b) \in \{t, t+1\} \times \{t, t+1\}$:

$$D_b(\mathbf{x}_a, \mathbf{y}_a; \mathbf{g}_a) = \sup\left\{\beta \in \mathbb{R} : (\mathbf{x}_a - \beta \mathbf{g}^i_a, \mathbf{y}_a + \beta \mathbf{g}^o_a) \in \boldsymbol{\mathcal{T}}_b\right\}, \tag{1}$$

if $(\mathbf{x}_a - \beta \mathbf{g}^i_a, \mathbf{y}_a + \beta \mathbf{g}^o_a) \in \boldsymbol{\mathcal{T}}_b$ for some β and $D_b(\mathbf{x}_a, \mathbf{y}_a; \mathbf{g}_a) = -\infty$ otherwise. Here, $\mathbf{g}_a = (\mathbf{g}^i_a, \mathbf{g}^o_a)$ represents the directional vector.

Chambers (2002) defines the Luenberger productivity indicator as:

$$\begin{aligned} &L_{t,t+1}(\mathbf{x}_t, \mathbf{y}_t, \mathbf{x}_{t+1}, \mathbf{y}_{t+1}; \mathbf{g}_t, \mathbf{g}_{t+1}) \\ &\quad = \frac{1}{2}\left[(D_t(\mathbf{x}_t, \mathbf{y}_t; \mathbf{g}_t) - D_t(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}; \mathbf{g}_{t+1}))\right. \\ &\quad \left. + (D_{t+1}(\mathbf{x}_t, \mathbf{y}_t; \mathbf{g}_t) - D_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}; \mathbf{g}_{t+1}))\right]. \end{aligned} \tag{2}$$

It can be decomposed in technical change and technical inefficiency change (Chambers et al., 1996), but the exact contribution of output and input change cannot be determined. This is because, in general, $\mathbf{g}_t = (\mathbf{g}^i_t, \mathbf{g}^o_t) > 0$ and inputs are contracted simultaneously as outputs are expanded in the directional distance functions (Ang and Kerstens, 2017a). Hence, it is not "additively complete" (O'Donnell, 2012). Furthermore, unlike the Luenberger-Hicks-Moorsteen (Briec and Kerstens, 2011), the Luenberger productivity indicator is not "determinate" in that it can be undefined (Briec and Kerstens, 2009).[1]

Furthermore, Chambers (2002) defines the output-quantity Luenberger productivity indicator as:

$$LO_{t,t+1}(\mathbf{x}_t, \mathbf{y}_t, \mathbf{x}_{t+1}, \mathbf{y}_{t+1}; \mathbf{g}^o_t, \mathbf{g}^o_{t+1}) = \frac{1}{2}\left[LO_t + LO_{t+1}\right] \tag{3a}$$

where the base period t output-profit indicator is defined as:

$$LO_t(\mathbf{x}_t, \mathbf{y}_t, \mathbf{y}_{t+1}; \mathbf{g}^o_t, \mathbf{g}^o_{t+1}) = D_t(\mathbf{x}_t, \mathbf{y}_t; (0, \mathbf{g}^o_t)) - D_t(\mathbf{x}_t, \mathbf{y}_{t+1}; (0, \mathbf{g}^o_{t+1})), \tag{3b}$$

[1]Determinateness of the indicator does not necessarily carry over to its components. This occurs for example in the empirical application of Ang and Kerstens (2017a), where the technical change component is undefined for one of the observations.

and the base period $t+1$ output-profit indicator:

(3c)

$$LO_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \mathbf{y}_t; \mathbf{g}_t^o, \mathbf{g}_{t+1}^o) = D_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_t; (0, \mathbf{g}_t^o)) - D_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}; (0, \mathbf{g}_{t+1}^o))$$

The input-quantity Luenberger productivity indicator:

(4a) $$LI_{t,t+1}(\mathbf{x}_t, \mathbf{y}_t, \mathbf{x}_{t+1}, \mathbf{y}_{t+1}; \mathbf{g}_t^i, \mathbf{g}_{t+1}^i) = \frac{1}{2}\left[LI_t + LI_{t+1}\right]$$

where the base period t input-profit indicator is defined as:

(4b) $$LI_t(\mathbf{x}_t, \mathbf{x}_{t+1}, \mathbf{y}_t; \mathbf{g}_t^i, \mathbf{g}_{t+1}^i) = D_t(\mathbf{x}_t, \mathbf{y}_t; (\mathbf{g}_t^i, 0)) - D_t(\mathbf{x}_{t+1}, \mathbf{y}_t; (\mathbf{g}_{t+1}^i, 0))$$

and the base period $t+1$ input-profit indicator:

(4c)

$$LI_{t+1}(\mathbf{x}_t, \mathbf{x}_{t+1}, \mathbf{y}_{t+1}; \mathbf{g}_t^i, \mathbf{g}_{t+1}^i) = D_{t+1}(\mathbf{x}_t, \mathbf{y}_{t+1}; (\mathbf{g}_t^i, 0)) - D_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}; (\mathbf{g}_{t+1}^i, 0))$$

The output-quantity (input-quantity) Luenberger productivity indicator $LO_{t,t+1}(\cdot)$ ($LI_{t,t+1}(\cdot)$) measures productivity solely in the output (input) directions. Thus, a combination of both $LO_{t,t+1}(\cdot)$ and $LI_{t,t+1}(\cdot)$ is "additively complete". This is Briec and Kerstens (2004)'s Luenberger-Hicks-Moorsteen productivity indicator (see Section 3 *infra*).

2.2. **Bennet indicators.** The preceding productivity measures have the advantage that they can be computed in the absence of price data, but their major drawback is that they require the approximation of the technology set and estimation of distance functions. This makes them somewhat harder to compute. We now focus our attention to productivity measures that are easy to compute using price data and which do not require estimation of distance functions.

Assume that the preceding distance functions can be estimated by a quadratic functional form at time h:

(5a) $$D_h(\mathbf{x}, \mathbf{y}; (\mathbf{g}^i, \mathbf{g}^o)) = a_h^0 + \sum_{u=1}^{n} a_h^u x^u + \sum_{k=1}^{m} b_h^k y^k + \frac{1}{2}\sum_{u=1}^{n}\sum_{v=1}^{n} \alpha_h^{uv} x^u x^v + \frac{1}{2}\sum_{k=1}^{m}\sum_{l=1}^{m} \beta_h^{kl} y^k y^l + \sum_{u=1}^{n}\sum_{k=1}^{m} \gamma_h^{uk} x^u y^k,$$

with the restrictions

(5b) $$\alpha_h^{uv} = \alpha_h^{vu}, \beta_h^{kl} = \beta_h^{lk},$$

(5c) $$\sum_{k=1}^{m} b_h^k g_k^o - \sum_{u=1}^{n} a_h^u g_u^i = -1;$$

(5d) $$\sum_{k=1}^{m} \gamma_h^{uk} g_k^o - \sum_{v=1}^{n} \alpha_h^{uv} g_v^i = 0, \qquad u = 1, \dots, n;$$

(5e) $$\sum_{l=1}^{m} \beta_h^{kl} g_l^o - \sum_{u=1}^{n} \gamma_h^{uk} g_u^i = 0, \qquad k = 1, \dots, m.$$

The output (input) directional distance function is defined by setting $\mathbf{g}^{i(o)} = \mathbf{0}^{n(m)}$. Chambers (2002) then defines the Bennet profit indicator (6a) by the difference between the Bennet revenue indicator (6b) and Bennet cost indicator (6c):

(6a)
$$BP(\mathbf{p}_t, \mathbf{p}_{t+1}, \mathbf{w}_t, \mathbf{w}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t+1}, \mathbf{x}_t, \mathbf{x}_{t+1}) = BR(\mathbf{p}_t, \mathbf{p}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t+1}) - BC(\mathbf{w}_t, \mathbf{w}_{t+1}, \mathbf{x}_t, \mathbf{x}_{t+1}).$$

with

(6b) $$BR(\mathbf{p}_t, \mathbf{p}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t+1}) = \frac{1}{2}\left[\mathbf{p}_t(\mathbf{y}_{t+1} - \mathbf{y}_t) + \mathbf{p}_{t+1}(\mathbf{y}_{t+1} - \mathbf{y}_t)\right],$$

and

(6c) $$BC(\mathbf{w}_t, \mathbf{w}_{t+1}, \mathbf{x}_t, \mathbf{x}_{t+1}) = \frac{1}{2}\left[\mathbf{w}_t(\mathbf{x}_{t+1} - \mathbf{x}_t) + \mathbf{w}_{t+1}(\mathbf{x}_{t+1} - \mathbf{x}_t)\right].$$

Avoiding any estimation procedure, these Bennet indicators are straightforward to compute from available data. Hence, it is of practical interest to establish the conditions under which the Luenberger productivity indicator can be computed by a Bennet profit indicator.

Proposition 1 (Theorem 6 in Chambers (2002))**.** *If firms maximize profit, and the technology directional distance function is quadratic with $\alpha_t^{ij} = \alpha_{t+1}^{ij}$ for all i and j, $\beta_t^{ij} = \beta_{t+1}^{ij}$ for all i and j then*

$$L_{t,t+1}(\mathbf{x}_t, \mathbf{y}_t, \mathbf{x}_{t+1}, \mathbf{y}_{t+1}; \mathbf{g}_t, \mathbf{g}_{t+1}) = BP(\hat{\mathbf{p}}_t, \hat{\mathbf{p}}_{t+1}, \hat{\mathbf{w}}_t, \hat{\mathbf{w}}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t+1}, \mathbf{x}_t, \mathbf{x}_{t+1})$$

where $\hat{\mathbf{p}}_k = \frac{\mathbf{p}_k}{\mathbf{p}_k \mathbf{g}_k^o + \mathbf{w}_k \mathbf{g}_k^i}$ *and* $\hat{\mathbf{w}}_k = \frac{\mathbf{w}_k}{\mathbf{p}_k \mathbf{g}_k^o + \mathbf{w}_k \mathbf{g}_k^i}$.

It turns out that the Bennet profit indicator is a superlative indicator of the Luenberger productivity indicator under an appropriate price normalization and when the directional distance function can be approximated by the quadratic functional form (5) with time-invariant second order coefficients. Thus, time can only affect the slope but not the curvature of the frontier.

3. Superlative approximation of Luenberger-Hicks-Moorsteen productivity

Briec and Kerstens (2004) define the LHM productivity indicator with base period t as the difference between the Luenberger output profit indicator and the Luenberger input profit indicator:[2]

$$\begin{aligned}(7)\quad & LHM_t(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \mathbf{x}_t, \mathbf{y}_t; \mathbf{g}_t, \mathbf{g}_{t+1}) \\ & = (D_t(\mathbf{x}_t, \mathbf{y}_t; (0, \mathbf{g}_t^o)) - D_t(\mathbf{x}_t, \mathbf{y}_{t+1}; (0, \mathbf{g}_{t+1}^o))) \\ & - (D_t(\mathbf{x}_{t+1}, \mathbf{y}_t; (\mathbf{g}_{t+1}^i, 0)) - D_t(\mathbf{x}_t, \mathbf{y}_t; (\mathbf{g}_t^i, 0))) \\ & \equiv LO_t(\mathbf{x}_t, \mathbf{y}_t, \mathbf{y}_{t+1}; \mathbf{g}_t^o, \mathbf{g}_{t+1}^o) - [-LI_t(\mathbf{x}_t, \mathbf{x}_{t+1}, \mathbf{y}_t; \mathbf{g}_t^i, \mathbf{g}_{t+1}^i)].\end{aligned}$$

The LHM productivity indicator with base period $t+1$ is:

$$\begin{aligned}(8)\quad & LHM_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \mathbf{x}_t, \mathbf{y}_t; \mathbf{g}_t, \mathbf{g}_{t+1}) \\ & = (D_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_t; (0, \mathbf{g}_t^o)) - D_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}; (0, \mathbf{g}_{t+1}^o))) \\ & - (D_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}; (\mathbf{g}_{t+1}^i, 0)) - D_{t+1}(\mathbf{x}_t, \mathbf{y}_{t+1}; (\mathbf{g}_t^i, 0))) \\ & \equiv LO_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \mathbf{y}_t; \mathbf{g}_t^o, \mathbf{g}_{t+1}^o) + LI_{t+1}(\mathbf{x}_t, \mathbf{x}_{t+1}, \mathbf{y}_{t+1}; \mathbf{g}_t^i, \mathbf{g}_{t+1}^i).\end{aligned}$$

One takes an arithmetic mean of LHM_t and LHM_{t+1} to avoid an arbitrary choice of base periods:

$$\begin{aligned}(9)\quad & LHM_{t,t+1}(\mathbf{x}_t, \mathbf{y}_t, \mathbf{x}_{t+1}, \mathbf{y}_{t+1}; \mathbf{g}_t, \mathbf{g}_{t+1}) \\ & = \frac{1}{2}\big[LHM_t(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \mathbf{x}_t, \mathbf{y}_t; \mathbf{g}_t, \mathbf{g}_{t+1}) \\ & + LHM_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \mathbf{x}_t, \mathbf{y}_t; \mathbf{g}_t, \mathbf{g}_{t+1})\big]\end{aligned}$$

Recently, Ang and Kerstens (2017a) show that the LHM productivity indicator is "additively complete" and, following Diewert and Fox (2017), provide a decomposition in the usual components of technical change, technical inefficiency change and scale inefficiency change under minimal assumptions of the technology set. However, a superlative approximation of the LHM productivity indicator is presently absent in the literature, which disallows easy computation of the LHM productivity indicator in practice. Our main result addresses this gap:

[2] We follow Chambers (2002)'s definition of the input profit indicator, swapping the places of Briec and Kerstens (2004)'s definition.

Proposition 2. *If firms maximize profit and the directional distance function is quadratic with $\alpha_t^{ij} = \alpha_{t+1}^{ij}$ for all i and j, $\beta_t^{ij} = \beta_{t+1}^{ij}$ for all i and j then*

$$\begin{aligned} LHM_{t,t+1}(\mathbf{x}_t, \mathbf{y}_t, \mathbf{x}_{t+1}, \mathbf{y}_{t+1}; \mathbf{g}_t, \mathbf{g}_{t+1}) &= BP(\tilde{\mathbf{p}}_t, \tilde{\mathbf{p}}_{t+1}, \mathbf{w}_t^*, \mathbf{w}_{t+1}^*, \mathbf{y}_t, \mathbf{y}_{t+1}, \mathbf{x}_t, \mathbf{x}_{t+1}) \\ &= BR(\tilde{\mathbf{p}}_t, \tilde{\mathbf{p}}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t+1}) - BC(\mathbf{w}_t^*, \mathbf{w}_{t+1}^*, \mathbf{x}_t, \mathbf{x}_{t+1}), \end{aligned}$$

where $\tilde{\mathbf{p}}_k = \frac{\mathbf{p}_k}{\mathbf{p}_k \mathbf{g}_k^o}$ *and* $\mathbf{w}_k^* = \frac{\mathbf{w}_k}{\mathbf{w}_k \mathbf{g}_k^i}$.

Proof. We can write

$$LHM_{t,t+1}(\cdot) = \frac{1}{2}\left[LO_t(\cdot) + LO_{t+1}(\cdot)\right] + \frac{1}{2}\left[LI_t(\cdot) + LI_{t+1}(\cdot)\right].$$

From Theorem 4 in Chambers (2002), it follows that

$$\frac{1}{2}\left[LO_t(\cdot) + LO_{t+1}(\cdot)\right] = BR(\tilde{\mathbf{p}}_t, \tilde{\mathbf{p}}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t+1})$$

if firms maximize revenue and technology is quadratic with $\beta_t^{ij} = \beta_{t+1}^{ij}$ for all i and j. From Theorem 2 in Chambers (2002), we know that

$$\frac{1}{2}\left[LI_t(\cdot) + LI_{t+1}(\cdot)\right] = -BC(\mathbf{w}_t^*, \mathbf{w}_{t+1}^*, \mathbf{x}_t, \mathbf{x}_{t+1})$$

if firms minimize costs and technology is quadratic with $\alpha_t^{ij} = \alpha_{t+1}^{ij}$ for all i and j. Simultaneous revenue maximization and cost minimization is profit maximization, which yields the desired result. □

The condition under which both Bennet profit indicators (cfr. $BP(\cdot)$ in Proposition 1 and Proposition 2) are equivalent follows immediately:

Corollary 1.

$$BP(\tilde{\mathbf{p}}_t, \tilde{\mathbf{p}}_{t+1}, \mathbf{w}_t^*, \mathbf{w}_{t+1}^*, \mathbf{y}_t, \mathbf{y}_{t+1}, \mathbf{x}_t, \mathbf{x}_{t+1})$$

with $(\mathbf{g}_k^i, \mathbf{g}_k^o) = (\frac{\tau}{n\mathbf{w}_k}, \frac{\tau}{m\mathbf{p}_k})$ *and*

$$BP(\hat{\mathbf{p}}_t, \hat{\mathbf{p}}_{t+1}, \hat{\mathbf{w}}_t, \hat{\mathbf{w}}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t+1}, \mathbf{x}_t, \mathbf{x}_{t+1})$$

with $(\mathbf{g}_k^i, \mathbf{g}_k^o) = (\frac{\tau}{2n\mathbf{w}_k}, \frac{\tau}{2m\mathbf{p}_k})$ *coincide for any* $\tau \in \mathbb{R}$.

Proof. Both Bennet profit indicators only differ by their price normalization which is parametrized by the direction vectors. Both price normalizations coincide when:

$$\begin{aligned} &(\tilde{\mathbf{p}}_k, \mathbf{w}_k^*) = (\hat{\mathbf{p}}_k, \hat{\mathbf{w}}_k) \\ \Leftrightarrow &\left(\frac{\mathbf{p}_k}{\mathbf{p}_k \mathbf{g}_k^o}, \frac{\mathbf{w}_k}{\mathbf{w}_k \mathbf{g}_k^i}\right) = \left(\frac{\mathbf{p}_k}{\mathbf{p}_k \mathbf{g}_k^o + \mathbf{w}_k \mathbf{g}_k^i}, \frac{\mathbf{w}_k}{\mathbf{p}_k \mathbf{g}_k^o + \mathbf{w}_k \mathbf{g}_k^i}\right) \end{aligned}$$

or when both denominators equal some $\tau \in \mathbb{R}$. For the LHS this holds when $(\mathbf{g}_k^i, \mathbf{g}_k^o) = (\frac{\tau}{n\mathbf{w}_k}, \frac{\tau}{m\mathbf{p}_k})$ and for the RHS this holds when $(\mathbf{g}_k^i, \mathbf{g}_k^o) = (\frac{\tau}{2n\mathbf{w}_k}, \frac{\tau}{2m\mathbf{p}_k})$. □

Briec and Kerstens (2004) show that if and only if the technology is (i) inversely translation homothetic in the direction of $\mathbf{g}$; and (ii) exhibits graph translation homotheticity in the direction of $\mathbf{g}$, then the LHM productivity indicator and the Luenberger output (input) productivity indicator coincide. An equivalent condition in terms of Bennet profit indicators is the following:

Corollary 2. *If firms are profit-maximizing and* $\mathbf{p}_k\mathbf{g}_k^o = \mathbf{w}_k\mathbf{g}_k^i$*, then*

$$BP(\tilde{\mathbf{p}}_t, \tilde{\mathbf{p}}_{t+1}, \mathbf{w}_t^*, \mathbf{w}_{t+1}^*, \mathbf{y}_t, \mathbf{y}_{t+1}, \mathbf{x}_t, \mathbf{x}_{t+1})$$

and

$$BP(\tilde{\mathbf{p}}_t, \tilde{\mathbf{p}}_{t+1}, \tilde{\mathbf{w}}_t, \tilde{\mathbf{w}}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t+1}, \mathbf{x}_t, \mathbf{x}_{t+1})$$

(or $BP(\mathbf{p}_t^*, \mathbf{p}_{t+1}^*, \mathbf{w}_t^*, \mathbf{w}_{t+1}^*, \mathbf{y}_t, \mathbf{y}_{t+1}, \mathbf{x}_t, \mathbf{x}_{t+1})$*) (locally) coincide.*

Proof. Trivial and follows directly from Corollary 8 in Chambers (2002). □

4. Empirical application

4.1. **Data description.** Employing the AMADEUS database, the empirical application focuses on an unbalanced sample of 5,018 Italian food and beverage companies (NACE rev. 1.1 code 15) for the years 1995 − 2007. This sector consists of micro (staff in full-time equivalents (FTE) from 10 to 20; 1,789 observations), small (staff in FTE from 20 to 50; 1864 observations), medium (staff in FTE from 50 to 250; 1,250 observations) and large (staff in FTE larger or equal to 250; 115 observations) firms. Firms of different size are quite well represented in our sample with the exception of the large firms. We distinguish one output and three inputs. The annual turnover is the output. The inputs include labor, materials and fixed assets. Price indexes of annual turnover, material and fixed assets are obtained using the EU KLEMS database. The wage is computed by the ratio of labor expense to labor quantity. The price indexes and wage are deflated to constant 1995 prices. The deflator is obtained from OECD (2018). Implicit quantities of material, fixed assets and annual turnover are calculated by the respective ratio of monetary value to price index. A full description of the data set can be found in Merlevede et al. (2015). We follow the same data cleaning procedure as described in Verschelde et al. (2016): first, we removed observations with one or more improbable inputs (i.e., employment costs, deflated tangible fixed assets or deflated material costs less than 1000 EUR) or outputs (i.e., deflated turnover less than 1000 EUR); and second, we removed observations per sector-year whose sector-year growth rate fell outside the [1%, 99%] of sector-year growth rates. This avoids extreme effects due to outliers and noise in the data. Table 1 shows the summary statistics of the eventual data set.

Statistic	Mean	St. Dev.	Min	25%	75%	Max
Annual turnover price index	0.900	0.041	0.848	0.869	0.912	1.000
Implicit annual turnover quantity (in €)	18,349,838	54,140,328	114,023.100	3,011,035	16,778,375	1,660,189,355
Wage (in € per full-time equivalent)	24,756	7,314	7,536	20,798	28,515	133,693
Labor quantity (in full-time equivalents)	55.037	165.163	10	16	54	6,160
Material price index	0.927	0.028	0.890	0.902	0.937	1.000
Implicit material quantity (in €)	11,778,903	31,762,994	1,270	1,569,180	10,370,713	872,412,992
Fixed asset price index	0.958	0.023	0.923	0.938	0.978	1.000
Implicit fixed assets quantity (in €)	4,119,113	9,087,706	1,114	710,514	4,306,673	236,184,000

Table 1. Summary statistics for Italian food and beverage companies, $1995 - 2007$. Price indexes and wage are deflated to constant 1995 prices. There are $5,018$ observations.

4.2. **Estimation procedure.** We empirically compare Luenberger and LHM indicators to their superlative approximations. The superlative approximations of the Luenberger and LHM indicator can be easily computed by the Bennet indicators defined in Propositions 1 and 2, respectively. Computing Luenberger and LHM indicators requires the estimation of directional distance functions. In line with (5), we use a quadratic functional form that includes a time shifter and firm size dummy variables in the intercept a_h^0:

$$
\begin{aligned}
D_t(\mathbf{x},\mathbf{y};(\mathbf{g}^i,\mathbf{g}^o)) = a^0 + \sum_{u=1}^{n} a^u x^u + \sum_{k=1}^{m} b^k y^k + \frac{1}{2}\sum_{u=1}^{n}\sum_{v=1}^{n} \alpha^{uv} x^u x^v \\
+ \frac{1}{2}\sum_{k=1}^{m}\sum_{l=1}^{m} \beta^{kl} y^k y^l + \sum_{u=1}^{n}\sum_{k=1}^{m} \gamma^{uk} x^u y^k + a^{time}(t-1995) \\
+ a^{small} d_t^{small} + a^{medium} d_t^{medium} + a^{large} d_t^{large}.
\end{aligned} \tag{10}
$$

The coefficients in this specification are all time-invariant and therefore satisfy the conditions from our theoretical results. Note that this specification is stricter than required by the theoretical results, because the linear terms are also time-invariant. Thus, time directly affects the frontier through the time shifter and the firm size dummy variables. We deterministically estimate (10) in line with Aigner and Chu (1968):

$$\min_{e_k \geq 0} \sum_{k=1}^{K} e_k \tag{11a}$$

$$\text{s.t. } e_k = (10) \qquad \forall k = 1,\dots,K \tag{11b}$$

$$(5b) - (5e) \tag{11c}$$

$$\partial D(\mathbf{x}_k,\mathbf{y}_k;(\mathbf{g}^i,\mathbf{g}^o))/\partial x^u \geq 0 \qquad \forall k = 1,\dots,K;\ \forall u = 1,\dots,n \tag{11d}$$

$$\partial D(\mathbf{x}_k,\mathbf{y}_k;(\mathbf{g}^i,\mathbf{g}^o))/\partial y^v \leq 0 \qquad \forall k = 1,\dots,K;\ \forall v = 1,\dots,m. \tag{11e}$$

The objective function and the first constraint fit the specified quadratic form to the data while minimizing inefficiency. The second constraint ensures compliance with the translation property for the considered directional vectors. The final two constraints impose strong disposability of the inputs and outputs, respectively (Chambers, 2002, D.4, page 753).[3] The lower bound restriction $e_k \geq 0$ ensures that $D_t(\mathbf{x},\mathbf{y};(\mathbf{g}^i,\mathbf{g}^o)) \geq 0$

[3]The estimated directional distance functions violate the monotonicity constraint for fixed assets in a very small number of observations: in 14 observations for the input-output directional distance function and in 2 observations for the output-oriented directional distance function. There were no violations for the input-oriented directional distance function.

for all observations. Following Färe et al. (2005), we divide the values of the observations by the corresponding mean and use $(\mathbf{g}^i, \mathbf{g}^o) = (\mathbf{1}_n, \mathbf{1}_m)$ as the directional vector.

The Luenberger and LHM indicators are computed by estimating the directional distance functions and plugging them into equations (2) and (7)-(9), respectively. The coefficients of (10) are shown in Table A1 in the Appendix. As expected, technical inefficiency decreases with firm size. For completeness, the Appendix also shows the histograms of inefficiency scores in Figure A1.

4.3. **Results.** Figure 1 shows the median LHM indicator and its superlative approximation by the Bennet indicator. The overall median LHM indicator is -0.0003, while the overall median Bennet indicator (BPLHM) is -0.003. Both overall median TFP indicators thus indicate a very slight decline. The left-hand figure illustrates that the values of the median Bennet indicator tend to be more extreme than those of the median LHM indicator. The median Bennet indicator peaks at $+0.034$ in $2003-2004$, while the highest value of the LHM indicator is $+0.023$ in the same period. The lowest value of the median Bennet indicator is -0.052 in $1996-1997$, while that of the median LHM indicator is -0.025 in the same period. Nonetheless, the annual median and trend of both indicators are relatively close. One observes in the right-figure that the the differences between the LHM and Bennet indicator are generally concentrated close to zero, with some extreme values.

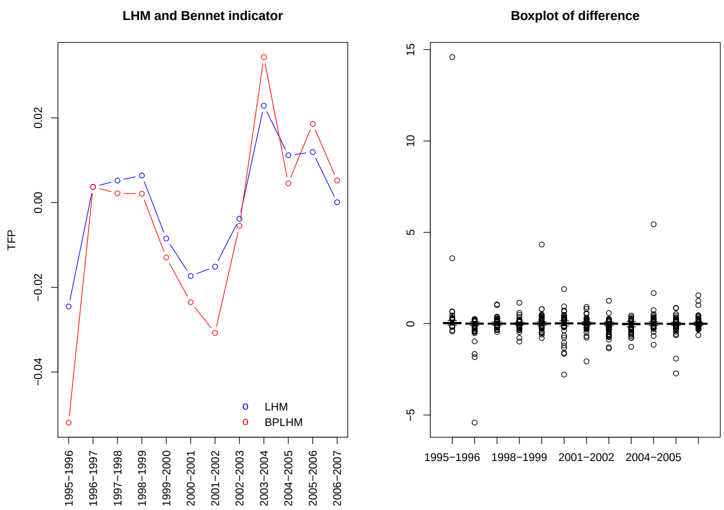


FIGURE 1. Median LHM indicator and its superlative approximation by the Bennet indicator.

Figure 2 shows the median Luenberger indicator and its superlative approximation by the Bennet indicator. The overall median Luenberger indicator is -0.004, while the overall median Bennet indicator (BPLuen) is -0.018. Remarkably, none of the median Bennet indicators exceed zero. In particular, the median Bennet indicator has plateaued to zero in $1996-1997$, $1997-1998$, $2003-2004$, $2004-2005$. $2005-2006$ and $2006-2007$. The lowest value of the median Bennet indicator is -0.073 in $1996 - 1997$. The value of the median Luenberger indicator ranges from -0.018 in $2000 - 2001$ to $+0.005$ in $2003-2004$. The left-hand figure shows that the troughs of the median Bennet indicator is much more extreme than those of the median Luenberger indicator. In contrast to the preceding comparison of the Bennet and LHM estimates, the current Bennet estimates follow a different trend than the Luenberger estimates. The right-figure shows that the the differences between the LHM and Bennet indicator are generally concentrated close to zero, with some few extreme values.

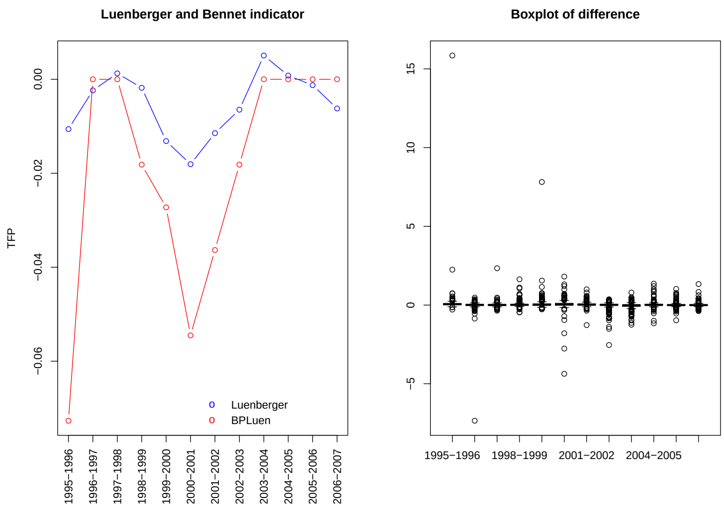
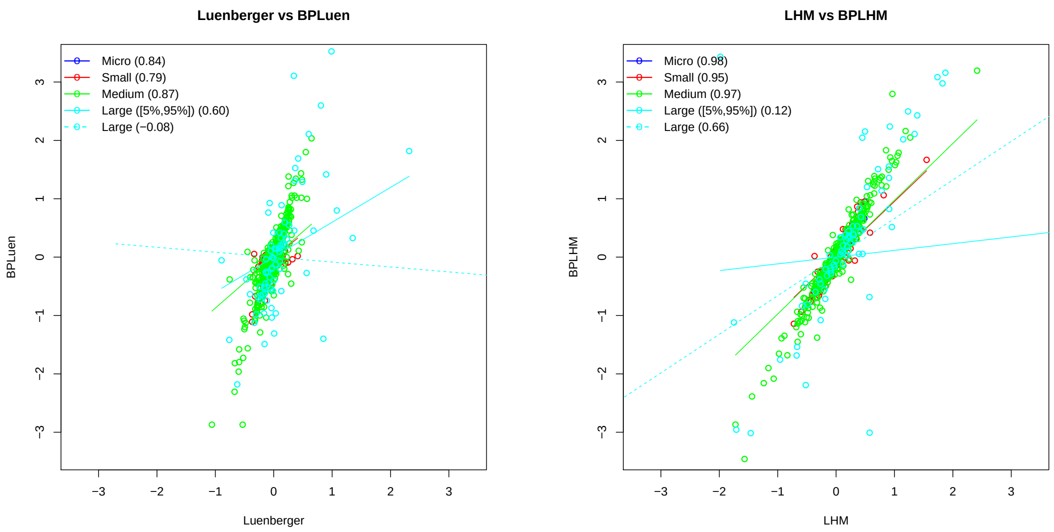


FIGURE 2. Median Luenberger indicator and its superlative approximation by the Bennet indicator.

Figure 3 compares the Luenberger (LHM) indicator to the respective Bennet indicator in the left-hand (right-hand) scatter plot by firm size. For non-large firms, the correlation between the Luenberger indicator and respective Bennet indicator is high. The correlations are 0.84, 0.79 and 0.87 for micro, small and medium firms, respectively. For large firms, there is substantial dispersion in values. Therefore, we present the results for all large firms, on the one hand, and those for the large firms excluding observations below the 5th and above the 95th percentile, on the other. For the former, the correlation is -0.08, while it is 0.60 for the latter. The correlation between the LHM indicator and Bennet indicator is even higher. The correlations are 0.98, 0.95, 0.97, 0.12 and 0.66 for micro, small, medium, large (excluding eight observations below the 5th and above the 95th percentile) and large firms (whole sample), respectively. Observe that the absolute values of Bennet estimates are generally higher than those of the corresponding Luenberger and LHM estimates, and that this especially holds for the latter. This is confirmed by table A2 in the Appendix, which shows the median productivity estimates by firm size and per year.



(A) Luenberger (B) LHM

Figure 3. Luenberger-type indicators vs. Bennet profit indicator by firm size.

Table 2 shows the Spearman rank correlation between the Luenberger indicator and respective Bennet indicator, on the one hand, and between the LHM indicator and respective Bennet indicator, on the other, by size and per year. Focusing on micro, small and medium firms, the rank correlations are generally high and significant at the 0.001 level for both comparisons throughout the whole observed period. The rank correlations between the LHM indicator and the respective Bennet indicator are consistently higher than those between the Luenberger and respective Bennet indicator. The rank correlations between LHM and Bennet indicators exceed 0.90 in all but two years. For large firms, the rank correlations fluctuate substantially, and are often not significant at the 0.001 level. This holds for both comparisons. These findings are in line with those obtained with the preceding correlations observed in the scatter plots.

		Luenberger and Bennet		LHM and Bennet	
Firm size	Period	ρ	P value	ρ	P value
Micro	1995-1996	0.78	0.0000	0.96	0
	1996-1997	0.79	0	0.92	0
	1997-1998	0.87	0	0.97	0
	1998-1999	0.90	0	0.98	0
	1999-2000	0.88	0	0.96	0
	2000-2001	0.89	0	0.96	0
	2001-2002	0.85	0	0.97	0
	2002-2003	0.90	0	0.97	0
	2003-2004	0.92	0	0.95	0
	2004-2005	0.62	0.0000	0.94	0
	2005-2006	0.82	0	0.98	0
	2006-2007	0.86	0	0.98	0
Small	1995-1996	0.70	0	0.89	0
	1996-1997	0.80	0	0.94	0
	1997-1998	0.76	0	0.94	0
	1998-1999	0.79	0	0.94	0
	1999-2000	0.86	0	0.97	0
	2000-2001	0.92	0	0.98	0
	2001-2002	0.89	0	0.96	0
	2002-2003	0.94	0	0.98	0
	2003-2004	0.86	0	0.99	0
	2004-2005	0.78	0	0.98	0
	2005-2006	0.74	0	0.98	0
	2006-2007	0.83	0	0.98	0
Medium	1995-1996	0.61	0.0000	0.80	0
	1996-1997	0.83	0	0.92	0
	1997-1998	0.79	0	0.94	0
	1998-1999	0.86	0	0.98	0
	1999-2000	0.87	0	0.97	0
	2000-2001	0.90	0	0.98	0
	2001-2002	0.74	0	0.94	0
	2002-2003	0.94	0	0.99	0
	2003-2004	0.94	0	0.98	0
	2004-2005	0.81	0	0.91	0
	2005-2006	0.76	0	0.99	0
	2006-2007	0.81	0	0.96	0
Large	1995-1996	-1	1	1	1
	1996-1997	-1	0.33	-0.50	1
	1997-1998	1	1	1	1
	1998-1999	0.40	0.75	0.80	0.33
	1999-2000	1	0.02	1	0.02
	2000-2001	0.68	0.05	0.98	0.0000
	2001-2002	0.94	0	0.95	0
	2002-2003	0.87	0.001	0.85	0.002
	2003-2004	0.86	0.02	1	0.0004
	2004-2005	0.12	0.78	0.95	0.0004
	2005-2006	0.82	0.01	0.98	0
	2006-2007	0.67	0.08	0.90	0.005

TABLE 2. Spearman rank correlation (ρ) between the Luenberger/LHM indicator and respective Bennet indicator, per year and firm size.

Finally, using a nonparametric test by Li et al. (2009), we annually compare the statistical distribution of the Luenberger (LHM) indicator to that of the respective Bennet indicator. The null hypothesis states that the compared distributions are the same over their entire support. We reject the null hypothesis at the 0.05 level that the empirical distribution of the Luenberger indicator is equal to that of the respective Bennet indicator for all but two periods ($1996-1997$ and $1997-1998$). Focusing on the distributional comparison of LHM and Bennet indicators, we cannot reject the null hypothesis at the 0.05 level for $1996-1997$, $1998-1999$, $2002-2003$, $2004-2005$ and $2005-2006$. Interestingly, we cannot reject the null hypothesis at the 0.001 level for the entire observed period. These results suggest that the statistical distribution of Bennet estimates is similar to that of the LHM estimates, but not so much to that of the Luenberger estimates.

	Luen = BPLuen		LHM = BPLHM	
Period	T_n	p-value	T_n	p-value
1995-1996	11.104	0	0.101	0.003
1996-1997	23.422	0.997	7.489	0.747
1997-1998	14.492	0.386	-8.779	0.003
1998-1999	-7.133	0	-14.063	0.153
1999-2000	1.331	0	-3.289	0.010
2000-2001	-15.558	0	-9.063	0.035
2001-2002	1.917	0	-26.863	0.043
2002-2003	-9.435	0	23.240	0.997
2003-2004	-2.707	0	-5.561	0.008
2004-2005	-6.366	0	-37.191	0.155
2005-2006	-4.412	0	-19.430	0.125
2006-2007	-9.536	0	0.217	0.005

TABLE 3. Results of Li test

5. Conclusions

This paper shows that the Bennet profit indicator is a superlative approximation of the LHM indicator when one can assume that there is profit-maximizing behavior and the input and output directional distance functions are quadratic in inputs and outputs. The Bennet profit indicator thus approximates both Chambers (2002)'s Luenberger indicator and Briec and Kerstens (2004)'s LHM indicator under equivalent conditions. This parallels Mizobuchi (2017)'s finding that the Törnqvist index is a superlative approximation of both Malmquist and Hicks-Moorsteen indexes under equivalent conditions. Our finding differs subtly in that our equivalence requires a different price normalization, which is not required for Mizobuchi (2017)'s finding. This is a direct consequence of the fact that ratio-based indexes, unlike difference-based indicators, are unit-invariant. We also show that the Luenberger- and LHM-approximating Bennet indicators coincide for an appropriate choice of directional vectors.

The empirical application focuses on Italian food and beverage companies for the years 1996 − 2007. It shows that the Bennet estimates are similar to the LHM estimates in terms of value and (Spearman rank) correlation, and relatively close in terms of statistical distribution. This only holds to a lesser extent for the Bennet estimates approximating the Luenberger estimates. For our application, the Bennet indicator is thus a simple yet empirically comparable alternative to the LHM indicator, but less so to the Luenberger indicator.

We have several recommendations for future research. First, components of productivity growth can be compared among the Luenberger and LHM indicators and their Bennet counterparts. Epure et al. (2011) and Ang and Kerstens (2017a) respectively show how Luenberger and LHM indicators can be decomposed into technical change, technical efficiency change and scale efficiency change. Ang (2018) develops a general framework to decompose all Bennet-type indicators, including the Bennet indicator, into technical change, technical efficiency change, scale efficiency change and mix efficiency change. Second, one could investigate which theoretical violations may deter superlative approximation. One way forward is a behavioral test of profit maximization along the lines of Varian (1984) and an in-depth analysis of the functional form and first- and second-order conditions. Third, we recommend to further search for superlative approximations of difference-based productivity indicators. While the literature on ratio-based approximations is rich, there seem to be only very few studies focusing on difference-based approximations. Finally, one could extend this framework to the dynamic context. The current framework is static in that it assumes that the level of inputs and outputs can be changed instantaneously to their optimum. It ignores the sluggish adjustment of quasi-fixed inputs and the intertemporal links between production periods. From this perspective, there are several interesting developments in the literature. Following Silva and Stefanou (2003), Ang and Oude Lansink (2018) show how to appropriately account for adjustment costs in the technology in a context of dynamic profit maximization. Similar in spirit to network DEA models (see Kao (2014) for a review), Cherchye et al. (2018) introduce a nonparametric framework for intertemporal cost minimization where intertemporal production links are modeled using durable and storable inputs.

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Appendix A. Supplementary tables and figures

	$D_t(\mathbf{x}, \mathbf{y}; (\mathbf{1}_n, \mathbf{1}_m))$	$D_t(\mathbf{x}, \mathbf{y}; (\mathbf{1}_n, \mathbf{0}_m))$	$D_t(\mathbf{x}, \mathbf{y}; (\mathbf{0}_n, \mathbf{1}_m))$
a_0	-0.060	-0.106	-0.149
a_L	0.326	0.629	0.814
a_M	0.344	0.350	1.150
a_F	0.030	0.022	0.131
α_{LL}	-0.008	-0.005	-0.003
α_{LM}	0.006	0.005	-0.005
α_{LF}	0.001	0.001	0.004
α_{MM}	-0.005	-0.005	-0.012
α_{MF}	-0.002	0.001	-0.003
α_{FF}	0.000	-0.001	-0.007
γ_{LY}	-0.001	-0.006	0
γ_{MY}	-0.000	0.006	0
γ_{FY}	-0.000	-0.001	0
b_Y	-0.300	-0.342	-1
β_{YY}	-0.001	-0.009	0
a_{time}	0.005	0.005	0.017
a_{small}	-0.014	-0.071	0.041
a_{medium}	-0.164	-0.351	-0.370
a_{large}	-1.228	-2.081	-2.668

Table A1. Coefficient estimates of the directional distance functions

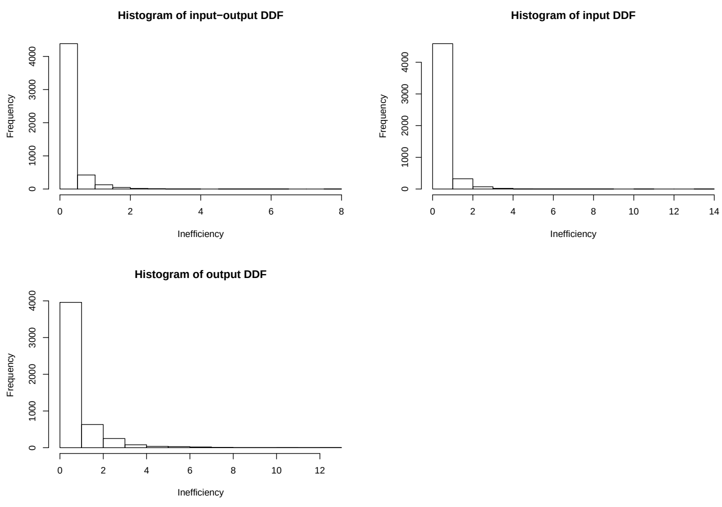

Figure A1. Histogram of inefficiency scores.

	1995-1996	1996-1997	1997-1998	1998-1999	1999-2000	2000-2001	2001-2002	2002-2003	2003-2004	2004-2005	2005-2006	2006-2007
Luen (Micro)	-0.009	-0.004	0.000	-0.005	-0.004	-0.013	-0.006	-0.007	-0.000	0.001	-0.001	-0.006
BPLuen (Micro)	-0.013	-0.003	0.004	0.003	-0.012	-0.019	-0.011	-0.007	0.005	0.000	0.000	-0.006
LHM (Micro)	-0.015	0.002	-0.006	-0.006	-0.055	-0.055	-0.026	-0.005	0.050	0.004	-0.004	-0.013
BPLHM (Micro)	4.253	-0.093	-0.050	0.036	-0.407	0.345	0.102	0.103	-0.143	-0.033	0.047	-0.085
Luen (Small)	-0.036	-0.000	-0.000	-0.018	-0.000	-0.036	-0.018	-0.018	-0.000	0.000	-0.000	-0.000
BPLuen (Small)	-0.073	-0.000	0.000	-0.000	-0.018	-0.045	-0.036	-0.018	0.018	-0.000	-0.000	-0.000
LHM (Small)	-0.127	-0.018	0.000	-0.036	-0.145	-0.145	-0.109	-0.018	0.164	-0.018	-0.000	-0.000
BPLHM (Small)	-4.796	0.763	-0.236	-0.191	-0.636	1.308	0.227	0.145	-0.545	-0.036	0.109	-0.454
Luen (Medium)	-0.008	-0.003	0.002	-0.002	-0.001	-0.009	-0.012	-0.007	-0.001	0.007	0.009	0.000
BPLuen (Medium)	-0.033	0.002	0.008	0.009	-0.008	-0.029	-0.018	-0.005	0.026	0.007	0.014	-0.003
LHM (Medium)	-0.046	0.038	0.003	0.035	-0.057	-0.061	-0.020	0.062	0.134	0.024	0.009	0.015
BPLHM (Medium)	6.102	0.186	0.148	0.336	0.165	0.921	0.034	0.313	0.117	0.303	0.266	-0.381
Luen (Large)	-0.022	-0.002	-0.000	-0.010	-0.002	-0.017	-0.023	-0.009	-0.002	0.006	0.011	0.003
BPLuen (Large)	-0.056	0.003	0.004	0.008	-0.014	-0.049	-0.034	-0.007	0.038	0.001	0.029	0.001
LHM (Large)	-0.118	0.031	-0.001	0.035	-0.087	-0.137	-0.053	0.104	0.224	0.001	0.016	0.052
BPLHM (Large)	-2.985	2.153	0.242	0.431	0.385	2.238	0.023	0.288	-0.123	0.336	0.365	-0.521

Table A2. Median TFP results by firm size