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Can Non-point Phosphorus Emissions from Agriculture be Regulated efficiently using Input-Output Taxes?

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Abstract

In many parts of Europe and North America, phosphorus loss from cultivated fields is threatening natural ecosystems. Though there are similarities to other non-point agricultural emissions like nitrogen that have been studied extensively, phosphorus is often characterised by the presence of large stocking capacities for phosphorus in farm soils and long time-lags between applications and emission. This makes it important to understand the dynamics of the phosphorus emission problem when designing regulatory systems. Using a model that reflects these dynamics, we evaluate alternative regulatory systems. Depending on the proportions of different types of farms in the agricultural sector, we find that an input-output tax system may be close to efficient, or in other cases must be supplemented with subsidy and manure reallocation schemes.

Keywords: regulating non-point pollution, phosphorus emissions, manure re-allocation, phosphorus stock dynamics.

JEL-codes: H23, Q1, Q5

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1. Introduction

Emissions of nutrients like nitrogen and phosphorus from cultivated fields can damage natural ecosystems in surrounding streams, lakes and inland sea areas[1]. While agricultural nitrogen emissions have been regulated intensively for several decades, recognition of the importance of agricultural phosphorus emissions is more recent. The environmental problems caused when ecosystems are overloaded with phosphorus are well known and point emissions from household sewage treatment and industry have been the focus of regulations for several decades in many countries. However, as emissions from point sources have been reduced, the relative importance of non-point agricultural emissions has increased. At the same time, the agricultural sector in many countries has seen the development of large farms specialising in intensive livestock production. These farms have been the source of absolute (sometimes dramatic) increases in phosphorus emissions (Maguire et al. 2009, Sharpley et al. 2003, Sharpley et al. 2009, Bundy et al. 2005). The main reason for this seems to be that it is profitable for intensive livestock farmers to apply manure to their fields in amounts that result in phosphorus applications well in excess of crop requirements (Ekholm et al. 2005). Initially this mainly causes phosphorous stocks in the field to build up, but as phosphorus stocks approach the fields' stocking capacity, leaching of phosphorus to the surroundings increases substantially. This process has led to soil-phosphorus accumulation (see Appendix A) and increased phosphorus run-off in many areas in Europe and North America (Sharpley et al. 2003, Sharpley 2009, Carpenter et al. 1998). Different types of regulations focusing specifically on phosphorus emissions have already been

[1] Nutrients such as phosphorus and nitrogen are essential for profitable crop and livestock agriculture and for natural ecosystems. However, phosphorus loss in eroded soil or through run-off from cultivated fields can damage water ecosystems by fuelling excessive algal growth and accelerating the eutrophication of lakes and streams. This can in turn reduce the benefits from other uses of these water resources such as fisheries, recreation, industrial uses and drinking water (Bundy et al. 2005, Sharpley et al. 2003, Carpenter et al. 1998).

implemented in a number of countries,[2] whilst addressing the problems caused by agricultural phosphorus emissions is becoming a high priority across the world.[3] With the increasing use of regulations aimed at reducing non-point agricultural phosphorus emissions, there is an increasing need for sound economic guidance about how to design such regulations cost-effectively. For this, two strands of literature seem relevant:

First the literature on non-point emissions offers insights into how to implement incentive corrections when regulators have incomplete information and cannot measure (non-point) emissions directly. A number of papers consider basing regulations on observable inputs and outputs. This has been suggested generally by Holtermann (1976) and specifically for nitrogen leaching by Huang and LeBlanc (1994), Fontein (1994), Helfand and House (1995), Fleming and Adams (1997), Hansen (1999) and Feinerman and Komen (2005). The basic idea is to establish a deposit-refund system for the core substance contained in the non-point emission, much like the deposit-refund systems that have been established for returnable bottles. When farmers pay a tax on the nitrogen content of their inputs and get a corresponding subsidy for the nitrogen content in their outputs, they are given an incentive to avoid leaching nitrogen through the production process. The idea of using input-output taxes also seems a feasible way of addressing the problem of non-observable phosphorus emissions[4]. However, the results from this literature do not carry over

[2] In the Netherlands, a mineral accounting system (MINAS) was implemented in 1998. If the phosphorus or nitrogen surplus exceeded a predefined limit, the mineral surplus was subject to a levy. In 2005, the MINAS system was replaced with a fertilisation balance approach where, e.g. application limits for animal manure and fertilisers were implemented (Oenema 2004, Oenema and Berentsen 2005). In Denmark, a tax on phosphorous contained in agricultural feed was introduced in 2005. Taxes on fertilisers have been implemented in several countries including Finland, Austria and Sweden (ECOTEC 2001, OECD 2008). In the United States, Agri-Environmental Policies (AEPs) have been in use since 1985 to reduce the negative environmental effects of agricultural production. Farmers are paid to reduce agri-environmental externalities such as soil erosion where the types of payments range from cost-sharing for specific conservation practices to incentives for the whole-farm management of environmental resources (Baylis et al. 2007, Hanrahan and Zinn 2005).

[3] For example the recent European Water Framework Directive makes the setting and implementation of goals for phosphorus emission from agriculture mandatory for all EU-countries.

[4]Other papers in the non-point pollution literature (e.g.Xepapadeas1992, Cabe and Herrings 1992, Horan et al. 1998, Hansen 1998, Smith and Tomasi 1999, Hansen 2002, Segerson and Wu 2006) investigate variations over

to phosphorus directly, because the literature does not consider the dynamics of large field stock capacities and long emission time lags[5].

The other strand of literature focuses specifically on phosphorous emissions and takes field stocks and long run dynamics into account (Schnitkey and Miranda 1993, Goetz 1997, Hediger 2003, Goetz and Keusch 2005, Iho 2007, Iho and Laukkanen 2009). These papers investigate the incentive corrections needed for arable farms using emission models that take account of the long time-lags between application and emission typical for phosphorus. For example, Iho and Laukkanen (2009) find that if incentives are not corrected, farmers will allocate too little land to buffer zones that reduce erosion and they will apply too much phosphorous fertiliser to their crops. However, these studies do not consider the problem of how to implement the needed incentive corrections when regulators have incomplete information, and most studies focus on arable production and do not consider the massive over-application of phosphorus on intensive livestock farms, which seems to be the core of the current problem. The only exception in this literature is an early study by Schnitkey and Miranda (1993) who develop a model with manure transportation costs that allows for rational farmers applying manure phosphorous in excess of agronomic recommendations.

This is our point of departure. In this paper, we develop a model of farm production that reflects the time dynamics of phosphorus emissions and allows farms with

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the ambient tax originally suggested by Segerson (1988) for regulating non-point emissions. However, the resulting tax payments may be perceived as unfair and politically unfeasible. At any rate, as far as we know, ambient taxes have not to date been applied in practice.

[5]A few studies have utilised detailed agricultural/natural science models of the dynamics of phosphorus (and nitrogen) flows and loss from agricultural land. Vatn et al. (1996, 1999, 2006) and Botterweg et al. (1998[a,b]) develop a very detailed bottom up ecological-economic farm level model describing and comparing different regulation scenarios with respect to phosphorus (and nitrogen) losses through erosion in two specific water catchments in Norway. Helin et al. (2006) use a similar approach to model nitrogen and phosphorus losses to a water catchment in South-Western Finland. Though these results are detailed and well founded, it is difficult to draw generally applicable conclusions from them about how to design regulations. Weersink et al. (2002) evaluate the effect of a phosphorus surplus tax on farm returns and herd size, but do not take stock effects into account.

intensive livestock production to rationally over-apply phosphorus to cultivated fields. Our main contribution is to introduce insights from the non-point literature, which allows us to design incentive systems for regulating non-point phosphorus emissions that are both feasible to implement when the regulator has incomplete information and that generate approximately optimal incentives.

What we do in the following is to combine the idea of addressing the non-observability of emissions through the use of input-output taxes (and other practically feasible bases for regulation) from the previous non-point literature with a dynamic model of phosphorus field stocks and livestock production, which incorporates the possibility of the surplus application of phosphorous suggested by Schnitkey and Miranda (1993). We find that a feasible input-output tax system can be close to efficient, when supplemented with subsidies for measures to reduce erosion and a manure reallocation credit scheme.

The paper is organised as follows. Section 2 presents phosphorus flows and stocks on a typical farm and develops the formal model. In section 3, we find the incentive corrections needed to implement Pareto optimum as our regulatory benchmark. In sections 4 and 5, we consider incentive regulation under optimistic and realistic assumptions about asymmetric information, whilst in section 6 we consider the regulation of a heterogeneous farm sector with arable, mixed and intensive livestock farms. Section 7 concludes the paper.

2. A model of agricultural phosphorus use and loss

Phosphorus is an essential nutrient for both arable and livestock production. The flows of goods and phosphorus through a typical farm are illustrated in figure 1 (See e.g. Hansen et al. 2010, Sharpley et al. 2003 or Bundy et al. 2005 for details). Arrows indicate flows where p_t^f is the flow of chemical phosphorous fertiliser input, x is a vector of other inputs, and y is a vector of output flows. When modelling in the following, we distinguish between the farm production system (*top box*) and the field's stock of phosphorous (*middle box*). Normally one would not make this distinction, but instead consider chemical phosphorous fertiliser as an input to farm production and disregard the field stock of phosphorous (i.e. implicitly assuming that it is a production process stock like the farmers stocks of feed and other inputs.). We make this distinction since we want to model the dynamics of the field phosphorous stock explicitly.

Phosphorus is imported to the farm production system (*top box*) in purchased inputs such as livestock feed (x_t). Letting α^x denote the vector of coefficients of phosphorous contained in the corresponding unit of the input vector *x*, the total amount of phosphorous imported through inputs is $\alpha^x x_t$. Phosphorus is exported again from the farm production system in outputs (y_t) such as crops, eggs, milk, meat, etc. where the total amount of phosphorous exported through outputs is $\alpha^y y_t$. Some of the phosphorous in livestock feed is digested and incorporated into the animals and eventually exported from the farm in livestock products. However, some of the phosphorous in livestock feed passes undigested through the animals and is incorporated into manure. Thus phosphorous is also exported from the farm production system to the farm field stock (*middle box*) when manure is applied to the fields (p_t^m)[6]. The amount of phosphorous in manure that is applied to the fields can be affected by farm decision variables like feed quality, feeding practices etc.

Finally, phosphorous is imported back into the farm production system when crops take up phosphorous from the farm field stock as they grow (p_t^c). Crop uptake of

[6] If the farm imports manure from other farms the element of the *x* vector indicating imported manure will be positive and if the farm exports manure the corresponding *y* vector element will be positive. Manure applied to field crops p_t^m equals manure produced on the farm plus manure imported from other farms less manure exported to other farms.

phosphorous is constrained by the amount of phosphorous available in the field stock. However, uptake may be *lower* than the available amount of phosphorous in the field stock if crop growth is constrained by other inputs like nitrogen fertilizer. From standard agronomics (Mengel et al. 2001), we know that crop growth requires nutrients (nitrogen and phosphorus) in fairly fixed proportions. This implies that the nutrient in shortest supply will constrain crop growth, while the surplus of the other nutrient will remain in the field. The crops phosphorus uptake (p^c) will there for be lower than the amount of phosphorous available in the field if crop growth is constrained by the amount of applied nitrogen fertiliser.

The *middle box* is the stock of phosphorous in the farm's fields (where P_t denotes field stock at the beginning of the growing season *t*). Phosphorous is added to the fields phosphorous stock when chemical fertiliser (p_t^f) and manure (p_t^m) are applied and the stock is reduced when crops take up phosphorous from the field (p_t^c). Basically, phosphorus applied to the fields that are not quickly taken up by crops is immobilized effectively and so can remain in the field for decades; even centuries (see e.g. Hansen et al. 2010 or Johnston et al. 2001 for a detailed description). However, phosphorous may be lost to the surrounding environment (*bottom box*) where it causes damage. This may happen during the process of applying manure and fertilizer before it reaches the field stock (application loss (p_t^a)).The proportion of phosphorous that is lost during application can be affected by the farmers timing (relative to rain and temperature) and his method of application (Sharpley et al. 2003, Johnsen 1993, Haygarth et al. 2009). In the diagram a_t is an indicator of the farmer's effort to reduce application loss.

Phosphorous may also leak from the fields stock through erosion (p_t^e). When field stocks are low field stock loss through erosion is limited. However, as phosphorus concentrations in the field increase, so will erosion losses. In addition, as the field-phosphorus stock approaches its maximum capacity for immobilising phosphorus, new surplus applications of phosphorus will not be immobilised quickly, but will instead remain dissolved so that additions to the field stock will quickly leach out through run-off (Sharpley et al. 2003). Erosion of the field stock can be influenced by the farmer through his cultivation practices (his choice of cover crops, contour farming tillage etc.) and through the size of uncultivated buffer zones (Sharpley et al. 2003, Carpenter et al. 1998). However, when maximum field stock capacity has been reached, buffer zones and cultivation practices (the farmer's efforts to reduce erosion loss, e_t) have little effect and the only way to reduce stock erosion loss is to reduce surplus application. In the diagram e_t is an indicator of the farmer's efforts to reduce field stock loss through erosion.

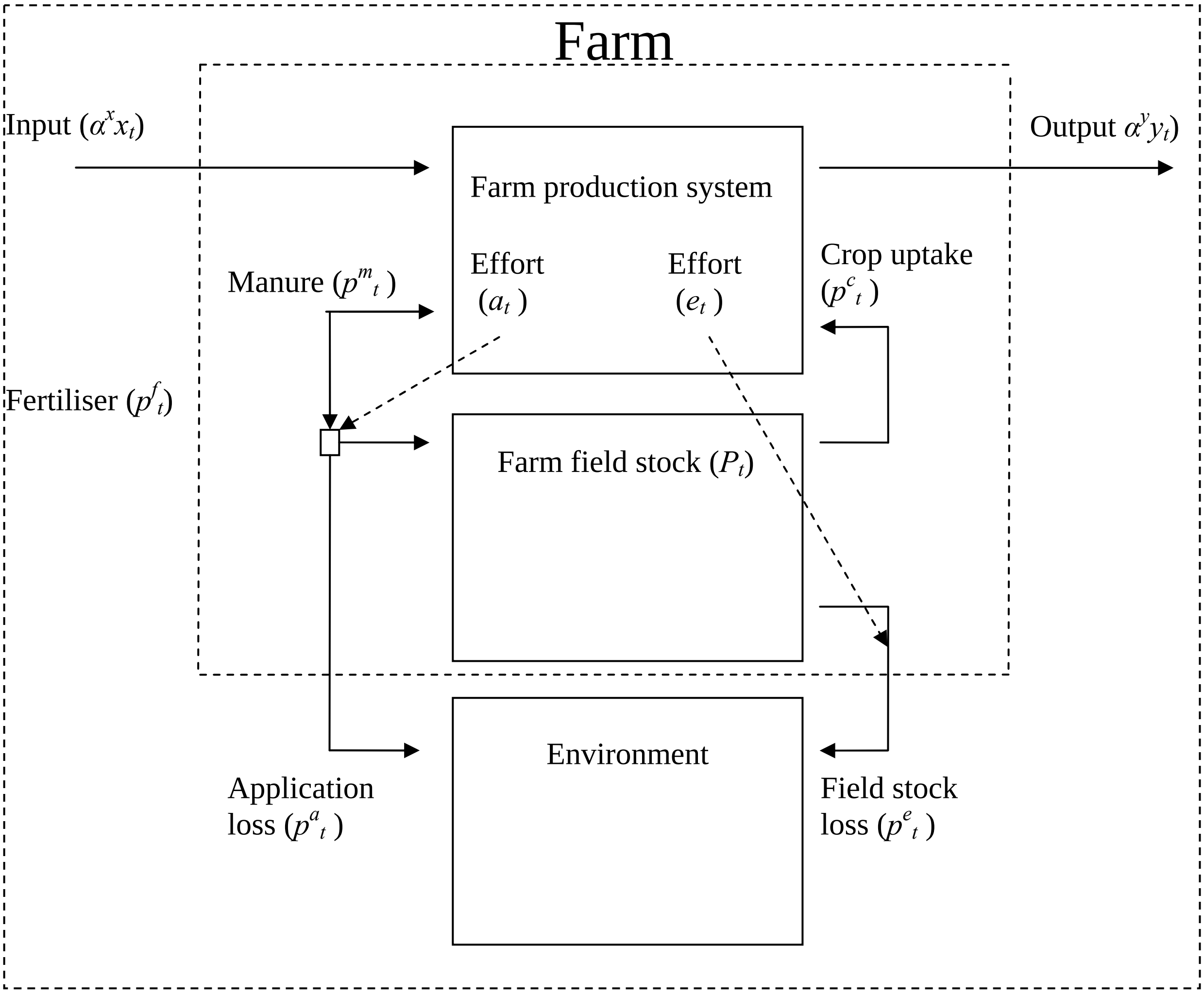

Figure 1 *Farm phosphorous stocks and flows and interaction with the environment and the market.*

The model

In the following our focus is on phosphorous dynamics so we represent the farm's production system (*top box*) as a profit function conditional on p_t^m, p_t^c, a_t, e_t :

$$\pi(p_t^m, p_t^c, a_t, e_t) \tag{1}$$

These four key variables are all affected by input-output and internal allocation decisions made by the farmer and so we let them be the farmer's explicit decision variables in (1). The function indicates the profit generated by the farms production system when all other decision variables (inputs, outputs, internal allocations etc.) are set at their profit maximising values conditional on these four key variables[7].

Crop uptake of phosphorous p_t^c is a farm decision variable (set indirectly by the farmer through his nitrogen fertilizer application) but the upper bound for this variable is the amount of phosphorous available in the field. Applied phosphorus, which is not lost during application or quickly taken up by crops, will be immobilised and stored in the soil where it may remain immobilised. Thus even if there is a substantial field stock of phosphorous, most of this is unavailable for plant uptake. Here we assume that only phosphorous which is applied during the growth season is available for uptake so that the following upper bound on crop uptake applies:

$$p_t^c \le p_t^m + p_t^f - p_t^a \qquad (2)$$

This is a simplification. In the short term (the time span of a few growing seasons), this constraint does not apply strictly. A small part of the phosphorous stock in the field is accessible to plants and can be mined for a few seasons (depending on the size of the

[7] Formally, let the farms production system be described as the production possibility set F of feasible production plans $(x_t, y_t, z_t, a_t, e_t, p_t^m, p_t^c)$, where z_t is a vector of all other farm decisions (internal allocations of inputs, internal allocation of the farmers effort etc.). Given input and output prices w^x, w^y the corresponding (dual) profit function is defined as:

$$\tilde{\pi}(w^x, w^y, a_t, e_t, p_t^m, p_t^c) = w^y y^* - w^x x^*$$

$$\text{where } (x_t^*, y_t^*, z_t^*) = \underset{x,y,z}{ArgMax}(w^y y - w^x x \,|\, (x_t, y_t, z_t, a_t, e_t, p_t^m, p_t^c) \in F)$$

To get a parsimonious representation we suppress input and output prices in the expression.

phosphorus stock). However, after this period, the farmer must again apply phosphorous corresponding to the plants' needs if he is to ensure continuing high yields. For example, the Danish Agriculture and Food Council recommends that farmers, under normal soil conditions, apply the constraint we assume for each growing season and so most farmers presumably do this (Rubæk et al. 2005, also see appendix A)

Since phosphorous cannot be created or destroyed by farm production, we also know that all feasible production plans by definition satisfy the mass balance condition (net import of phosphorous to the farm through inputs and outputs must equal net export to the farms field stock):

$$\alpha^x x_t - \alpha^y y_t = p_t^m - p_t^c \qquad (3)$$

This is useful because we will investigate regulation through taxes on inputs and outputs in the following. Equation (3) allows us to derive the incentives generated by such taxes for our explicit decision variables in (1).

Remembering that a_t denotes the farmer's effort to reduce application loss, we define application losses, p_t^a, as a function of applied manure and fertilizer volumes and this effort[8]:

$$p_t^a = p_t^a(p_t^m, p_t^f, a_t) \qquad (4)$$

[8] Were we assume positive derivatives for p_t^m, p_t^f and a negative derivative for a_t .

The erosion of field soil and phosphorous p_t^e is affected by the farmer's efforts to reduce erosion loss (e_t) and depends on how close the field-phosphorus stock at the beginning of the season (P_t) is to its maximum capacity for immobilising phosphorus (see above):

$$p_t^e = p_t^e(P_t, e_t) \qquad (5)$$

Here we assume that $\beta < \frac{\partial p_t^e}{\partial P_t} \leq 1$ (where $\beta > 0$ may be arbitrarily close to zero), $0 \leq \frac{\partial^2 p_t^e}{\partial^2 P_t}$ and that there exists a $\bar{P}$ so that $\frac{\partial p_t^e}{\partial P_t} = 1$ for $P_t \geq \bar{P}$. This captures the idea that the field has a limited capacity to stock phosphorus so that when the stock is equal to or larger than $\bar{P}$ additions to the stock during one season are emitted completely by the end of the following season through run off. If, on the other hand, stocks are small ($P_t << \bar{P}$), the emission of additions to the stock in one season will be stretched over a long time span and so the bulk of the addition will be retained in the field for many seasons (note however that $\beta > 0$ implies that though retention may be arbitrarily close to 1, emissions never completely disappear[9]).

By definition, total change in phosphorus stock over the season is the sum of stock additions (through manure and chemical fertiliser) minus stock reductions through crop uptake and phosphorous loss:

[9]This is a technical assumption that we need below, but since $\beta > 0$ can be arbitrarily close to zero it is not restrictive since generally there is always some field soil erosion.

$$p_t = p_t^m + p_t^f - p_t^c - p_t^a - p_t^e \qquad (6)$$

Remembering that P_t measures stock at the beginning of the season, this implies that the stock of phosphorus in the field develops over time according to:

$$P_{t+1} = P_t + p_t \qquad (7)$$

By successively inserting all prior periods' stock equations (7) into the current period's stock equation implies, that the current stock is a function of the initial stock in period 0 and all subsequent stock changes i.e.:

$$P_t = P_0 + \sum_{\tau=0}^{t-1} p_\tau \qquad \text{for } t=1....\infty \qquad (8)$$

Finally, we assume constant marginal damage of phosphorus emissions δ in the water environment, i.e.

$$D_t = \delta(p_t^a + p_t^e) \qquad (9)$$

Where D_t is environmental damage of emissions in period t . This implies that the damaged ecosystem is large compared to the individual farm. We are also assuming that the damage caused by emission does not vary over time. This is a useful assumption when we, in the following, want to understand the structure of the regulation problem. However, the assumption is not necessarily correct. Marginal ecosystem damage depends on aggregate emissions to the system as well as the accumulated stock of phosphorus in the ecosystem, both of which may change over time. We will discuss the implications of relaxing this assumption at the end of the paper.

The unregulated profit maximising farmer

We assume that the farmer maximises profit. To get total profit during season *t*, phosphorous fertiliser costs must be deducted from the production system profit, $\pi(p_t^m, p_t^c, a_t, e_t)$, i.e.:

$$\Pi_t = \pi(p_t^m, p_t^c, a_t, e_t) - w^f p_t^f \tag{10}$$

where w^f is the price of chemical phosphorous fertiliser. Since the farmer is constrained by (2), he does not mine his field stock of phosphorous and he is therefore unconcerned about the field stock of phosphorous or erosion from it. This implies that he can solve each period's maximisation problem independently of other periods. His only concern is the current constraint on plant available phosphorous in his fields (2). He can shift this constraint by applying manure and fertiliser and by avoiding application run off (4). In each period, the unregulated farmer chooses his decision variables $(p_t^m, p_t^c, p_t^f, a_t, e_t)$ so as to maximise current period profit, i.e.:

$$\underset{p_t^m, p_t^c, p_t^f, a_t, e_t}{Max} \quad \Pi_t \tag{11}$$
$$s.t. \quad (2),(4), p_t^m \geq 0, p_t^f \geq 0$$

We assume interior solutions for p_t^c, a, e but allow corner solutions for p_t^m, p_t^f. This allows our model to apply for arable farms (where $p_t^m = 0$), mixed farms (where $p_t^m > 0, p_t^f > 0$) and for farms on which livestock production is so intensive that the application of chemical phosphorous fertiliser is unnecessary ($p_t^f = 0$)[10]. There are two cases depending on whether the phosphorous constraint (2) is binding or not.

The first case captures a 'balanced' farm where phosphorous is not applied in excess of plants' needs ($p_t^c = p_t^m + p_t^f - p_t^a$). If the phosphorous constraint is binding, both constraints (2 and 4) can be inserted in the object function. First order conditions for the remaining decision variables are found by differentiation in the usual way:

$$
\begin{aligned}
&\frac{d\Pi_t}{dp_t^f} = -w^f + \frac{\partial \pi}{\partial p_t^c}\frac{dp_t^c}{dp_t^f} \le 0 \\
&\frac{d\Pi_t}{dp_t^m} = \frac{\partial \pi}{\partial p_t^m} + \frac{\partial \pi}{\partial p_t^c}\frac{dp_t^c}{dp_t^m} \le 0 \\
&\frac{d\Pi_t}{da_t} = \frac{\partial \pi}{\partial a_t} + \frac{\partial \pi}{\partial p_t^c}\frac{dp_t^c}{dp_t^a}\frac{\partial p_t^a}{\partial a_t} = 0 \qquad (12) \\
&\frac{d\Pi_t}{de_t} = \frac{\partial \pi}{\partial e_t} = 0
\end{aligned}
$$

In this case $\frac{\partial \pi}{\partial p_t^c} > 0$ and applied phosphorous that is not lost through application run off shifts the constraint (2) and is taken up by crops (i.e. $\frac{dp_t^c}{dp_t^m} = 1 - \frac{\partial p_t^a}{\partial p_t^m}$ and $\frac{dp_t^c}{dp_t^f} = 1 - \frac{\partial p_t^a}{\partial p_t^f}$). The first condition in (12) tells us that if chemical fertiliser is applied ($p_t^f > 0$), the cost must be equal to the marginal profit of shifting the crop phosphorous constraint in optimum. If the cost is greater than the marginal profit, chemical fertiliser is not applied, i.e. $p_t^f = 0$. This may be the case on farms with livestock production and ample supplies of phosphorous in

[10] Strictly speaking we should allow a corner solution for e but to keep things manageable we assume an interior solution. This is not important for results or interpretations.

manure. The term $\partial\pi / \partial p_t^m$ reflects the marginal profit of increasing the phosphorous content of manure, e.g. by adjusting the livestock feed. On farms with livestock production, $\partial\pi / \partial p_t^m$ is typically positive for small p_t^m values, where increasing phosphorus in livestock feed allows substantial increases in the production of meat, milk and other outputs per unit phosphorus increase in manure. As phosphorous contained in feed increases, livestock utilisation decreases and at some point the increase in feeding costs balances the increase in output revenue. After this point, much of the phosphorus in feed is passed on to manure and marginal profit ($\partial\pi / \partial p_t^m$) becomes negative. In optimum, the negative marginal profit $\partial\pi / \partial p_t^m$ just balances the value of shifting crops' phosphorus constraint (on arable farms without livestock where $p_t^m = 0$ the inequality applies). The third condition implies that the cost of effort to reduce application run off must equal the marginal value of the conserved phosphorous in crop production. Finally, the last equation indicates that effort to reduce the erosion of the field's phosphorous stock is increased until marginal profit is zero.

If the phosphorous constraint (2) is not binding (implying over application of phosphorous in manure to the fields, $p_t^c < p_t^m + p_t^f - p_t^a$), then in addition to (12), the first order condition $\frac{d\Pi_t}{dp_t^c} = \frac{d\pi}{dp_t^c} = 0$ applies and crop uptake is no longer a function of available phosphorous in the field and so:

$$\frac{dp_t^c}{dp_t^f} = \frac{dp_t^c}{dp_t^m} = \frac{dp_t^c}{dp_t^a} = 0 \qquad (13)$$

When inserting this in (12), the first condition becomes a corner solution where $p_t^f = 0$. This implies that when manure phosphorous is over applied, it is not efficient to apply costly chemical phosphorous fertiliser. The second condition shows that the marginal value of manure phosphorous is zero, whilst the third condition shows, as a consequence, it is not profitable to expend marginal costs to reduce application run off. This captures unbalanced farms with intensive livestock production. Here livestock production is so profitable that even when the marginal value of phosphorous in manure is zero, the profit maximising amount in manure is greater than can be profitably taken up by crops. As a result, manure phosphorous in effect becomes a by-product or a waste product of animal production that the farmer ‘discards’ on his field. It is this type of farm that seems to be the main cause for concern today.

The model allows for the export of manure from the farm as an output sold to other farmers. Hidden in the profit function, the sales price of phosphorous in the manure delivered to the purchasing farm will be the alternative fertiliser cost w^f. If transport was costless, this value would apply on all farms in the economy. However, if transport costs are substantial, the marginal profit from selling manure phosphorous may be substantially below w^f and may differ between farms, depending on distances to neighbouring farms and the concentration of intensive livestock farms in the area. For intensive livestock farms in areas where many other farms are of the same type, the local market for manure may even be saturated, in the sense that the marginal profit from sale is zero.

3. The regulatory benchmark

We assume that the regulator wishes to maximise the discounted sum of farm profits and environmental costs resulting from farming. As a benchmark, consider a situation in which the regulator has full knowledge and full control of all farm decision variables. In this case, the regulators welfare maximisation problem would be:

$$\underset{p_t^m, x_t^a, x_t^e, p_t^f, p_t^c}{Max} SW = \sum_{t=0}^{\infty}\left[\left(\pi(p_t^m, p_t^c, a_t, e_t) - w^f p_t^f\right) - \delta(p_t^a + p_t^e)\right](1+\rho)^{-t} \quad (14)$$

s.t. (2), (4), (5), (6) and (8) for t=0.... ∞

where ρ is the regulator's discount rate.

A change in the net flow of phosphorus in period t affects the next period's stock and by definition $\frac{dP_{t+1}}{dp_t} = 1$. The partial derivative for the following period stock is also by definition one, $\frac{\partial P_{t+2}}{\partial p_t} = 1$. However, since erosion loss p_t^e is a function of current stock the change in stock in period $t+1$ will affects erosion emission in period $t+1$ i.e. $\frac{dp_{t+1}^e}{dP_{t+1}}\frac{dP_{t+1}}{dp_t}$ and so the total derivative becomes: $\frac{dP_{t+2}}{dp_t} = \frac{\partial P_{t+2}}{\partial p_t} - \frac{dp_{t+1}^e}{dP_{t+1}}\frac{dP_{t+1}}{dp_t} = 1 - \frac{dp_{t+1}^e}{dP_{t+1}}$. Intuitively the stock increase is reduced in each period by the amount of phosphorus lost through erosion, which is caused by the stock increase from the last period. Under the assumed regularity conditions for the erosion constraint in (5), the proportion of a stock increase lost through erosion in any year is always strictly greater than β (which by definition is always greater than 0). This implies that $\frac{dP_T}{dp_t} \to 0$ for $T \to \infty$, and so eventually, all of the original stock increase is lost as erosion emission: $\sum_{\tau=t+1}^{T}\frac{\partial p_\tau^e}{\partial P_\tau}\frac{\partial P_\tau}{\partial p_t}\rightarrow 1$ for $T\rightarrow\infty$. Even though this may take a very long time, any increase in stock in a period *t* will eventually be lost through the erosion process.

Now consider the corresponding discounted sum of damage caused by these future erosion emissions: $0\leq\sum_{\tau=t+1}^{T}\frac{\partial p_\tau^e}{\partial P_\tau}\frac{\partial P_\tau}{\partial p_t}\delta(1+\rho)^{-\tau}=\delta\sum_{\tau=t+1}^{T}\frac{\partial p_\tau^e}{\partial P_\tau}\frac{\partial P_\tau}{\partial p_t}(1+\rho)^{-\tau}$ where :

$$0\leq\sum_{\tau=t+1}^{T}\frac{\partial p_\tau^e}{\partial P_\tau}\frac{\partial P_\tau}{\partial p_t}(1+\rho)^{-\tau}\rightarrow\gamma\leq 1 \text{ for } T\rightarrow\infty \tag{15}$$

In the following, we will use the limit value of this sum (γ) extensively as a simple way of summarising how additions to current phosphorus stocks affect future emission damage that concern the regulator. Clearly γ depends on the discount rate, on the current phosphorus stock P_t, and the speed with which it is increasing over time. When there is no discounting ($\rho=0$), $\gamma=1$ reflects that the regulator has the same concern about emissions in the distant future as current emissions. If the regulator discounts the future, ($\rho>0$), then $\gamma<1$. If the current stock is small and only slowly increasing so that emissions are substantially delayed, γ may be close to zero (which reflects that additions to the stock have little or no negative effect on the regulator's objective function because he has little concern for environmental effects which occur in the distant future). If on the other hand, current stock is close to $\overline{P}$, or is increasing fast so that it quickly will be close to $\overline{P}$, then $\frac{\partial p_\tau^e}{\partial P_\tau}\approx 1$, and most of the stock increase will be emitted quickly. Thus, for farms with stocks which are increasing rapidly, or stocks which are close to $\overline{P}$, $\gamma\approx 1$ irrespective of the discount rate.

Model solution

If the farmer does not have manure in excess of the phosphorous constraint, (4) is binding in (14) so that all constraints can be inserted in the object function and as for (12), first order conditions for the remaining decision variables are found by differentiation in the usual way:

$$\begin{aligned}
\frac{dSW}{dp_t^f} &= -w^f + \frac{\partial\pi}{\partial p_t^c}\frac{dp_t^c}{dp_t^f} - \delta\left(\frac{dp_t^a}{dp_t^f} + \left(1 - \frac{dp_t^c}{dp_t^f} - \frac{\partial p_t^a}{\partial p_t^f}\right)\gamma\right) = 0 \\
\frac{dSW}{dp_t^m} &= \frac{\partial\pi}{\partial p_t^m} + \frac{\partial\pi}{\partial p_t^c}\frac{dp_t^c}{dp_t^m} - \delta\left(\frac{dp_t^a}{dp_t^m} + \left(1 - \frac{dp_t^c}{dp_t^m} - \frac{\partial p_t^a}{\partial p_t^m}\right)\gamma\right) = 0 \\
\frac{dSW}{da_t} &= \frac{\partial\pi}{\partial a_t} + \frac{\partial\pi}{\partial p_t^c}\frac{dp_t^c}{dp_t^a}\frac{\partial p_t^a}{\partial a_t} - \delta\left(\frac{\partial p_t^a}{\partial a_t} - \frac{\partial p_t^a}{\partial a_t}\left(1 + \frac{dp_t^c}{dp_t^a}\right)\gamma\right) = 0 \\
\frac{dSW}{de_t} &= \frac{\partial\pi}{\partial e_t} - \delta\left(\frac{\partial p_t^e}{\partial e_t} - \frac{\partial p_t^e}{\partial e_t}\gamma\right) = 0
\end{aligned} \qquad (16)$$

where $\gamma = \sum_{\tau=t+1}^{\infty} \frac{\partial p_\tau^e}{\partial P_\tau}\frac{\partial P_\tau}{\partial p_t}(1+\rho)^{-\tau}$ as defined in (15)[11] and $\frac{dp_t^c}{dp_t^m} = 1 - \frac{\partial p_t^a}{\partial p_t^m}$, $\frac{dp_t^c}{dp_t^f} = 1 - \frac{\partial p_t^a}{\partial p_t^f}$ if the farm is phosphorous constrained, and $\frac{dp_t^c}{dp_t^m} = \frac{dp_t^c}{dp_t^f} = 0$ if the farm is unconstrained.

These are the first order conditions for the regulator's welfare maximisation problem where environmental damage is taken into account. Notice that without these damages (if δ =0), the regulators problem (14) becomes equivalent to the profit maximisation problem that the unregulated farmers solve (12).

The first condition reflects the private costs and benefits of chemical fertiliser application. The last element is the marginal environmental costs caused by applying chemical phosphorous fertiliser. The element $\frac{\partial p_t^a}{\partial p_t^f}$ in the parenthesis is the proportion of applied

[11]See appendix B for the differentiation details.

fertiliser lost immediately through application run off. Since this emission happens in the current period, the damage effect is not discounted. The next element $(1-\frac{dp_t^c}{dp_t^f}-\frac{\partial p_t^a}{\partial p_t^f})$ is the proportion of applied fertiliser that is added to the field stock (i.e. the part of applied fertiliser that is not lost through application run off or taken up by crops). This addition to the field stock is eventually lost to the environment, but since this does not happen immediately, the damage effect is discounted with the γ factor. As discussed above, if there is no discounting, $\gamma = 1$, since all phosphorus left on the field and incorporated into the stock will eventually be emitted. Thus γ indicates the 'environmental value' (due to regulator discounting) of delaying emissions through stockpiling on the farmer's field. In the same way, the second condition reflects the private costs and benefits of manure application and the marginal environmental costs hereof. The third condition captures the private and environmental effects of investing effort in reducing application run off. Finally, the last condition captures the environmental value of investing effort in reducing erosion from the phosphorous stock. As in (12), if the phosphorous constraint (4) is not binding (farmers producing manure as a waste product), then (16) still applies, but crop uptake is no longer a function of application (i.e. (13) applies).

Because of the time lag from over-application to emission, which is caused by the fields' capacity to stock phosphorous, the environmental effects comprise an important temporal dimension of the maximisation problem. If the regulator discounts future environmental costs, delaying emissions is valuable. This implies that the over-application of phosphorous may be much more harmful if it takes place on a farm where the resulting emissions occur faster than if the same over-application occurs on a farm where the resulting emissions arise in the distant future. If regulations are to be efficient, they must take account of this heterogeneity in resulting damages.

4. Regulating a farm under asymmetric information – the optimistic case

The regulator's problem is to induce farmers to take account of environmental damage from phosphorous emissions when they maximise profit. However, the regulator has limited knowledge and enforcement capacity. In the following, we assume that the regulator does not know the farmer's profit/production functions, nor can he measure the non-point emissions of phosphorus directly. Instead we assume that he can measure (and tax) the phosphorus content of farm inputs (such as feed and fertiliser) and outputs (such as crops, milk and meat). This seems feasible, at least if such taxes are not differentiated between farms (an issue we address in section 7). In this section, we also assume that the regulator can make accurate measurements of the phosphorus stock in the field. Regulators are able to make such estimates, but it requires on-farm inspection and comprehensive laboratory testing of soil samples if such estimates are to be exact. Thus this assumption is probably optimistic.

We assume that the farmer knows his own profit/production function. Without regulations, farmers are unconcerned about emissions of phosphorus from their fields or how current surplus application will affect such emissions in the future, i.e. the farmer has no reason to acquire knowledge of the erosion emission equation (5). The tax system implemented by the regulator gives the farmer such an incentive. In the following, we let $\frac{\partial p_\tau^e}{\partial P_\tau}^F$ and $\frac{\partial P_\tau}{\partial p_t}^F$ denote the farmers' estimates of $\frac{\partial p_\tau^e}{\partial P_\tau}$ and $\frac{\partial P_\tau}{\partial p_t}$ in equation (15). Further, let ρ^F denote the farmer's discount rate, which may differ from the regulator's discount rate.

Using the same arguments as for (15), we assume that the farmer is able to deduce that $\sum_{\tau=t+1}^{T} \frac{\partial p_\tau^{e\ F}}{\partial P_\tau} \frac{\partial P_\tau}{\partial p_t}^{F} \rightarrow 1$ for $T \rightarrow \infty$ so that:

$$\sum_{\tau=t+1}^{T} \frac{\partial p_\tau^{e\ F}}{\partial P_\tau} \frac{\partial P_\tau}{\partial p_t}^{F} (1+\rho^F)^{-\tau} \rightarrow \gamma^F \leq 1 \text{ for } T \rightarrow \infty \qquad (17)$$

Assuming that the regulator can measure the phosphorus content of inputs to and outputs from the farm, by using the mass balance equation (2), he can implement a tax on the surplus application of phosphorus to the farm fields ($p_t^m + p_t^f - p_t^c$). Let T_1 be the tax rate applied to this tax base. Further, assuming that the regulator can accurately measure the stock of phosphorus in the fields, he can calculate and tax the annual growth in the phosphorus field stocks ($P_{t+1} - P_t$). Let T_2 be the tax rebate rate applied to this tax base (reflecting that the farmer is refunded tax for the part of the surplus application that is incorporated into field stocks). In this way, only farmers with a high phosphorus stock in their fields continue to pay a high tax on net import of phosphorous – reflecting that these imports are quickly emitted to the environment from the field stock. With this tax system, the farmer's profit maximisation problem becomes (18):

$$\underset{p_t^m, x_t^a, x_t^e, p_t^f, p_t^c}{Max} \Pi = \sum_{t=0}^{\infty} \Big[\Big(\pi(p_t^m, p_t^c, a_t, e_t) - w^f p_t^f \Big) - \Big(T_1(p_t^m + p_t^f - p_t^c) - T_2(P_{t+1} - P_t) \Big) \Big] (1+\rho^F)^{-t}$$

s.t. (2), (4), (5), (6) and (8) for t=0....∞

with the following first order conditions (19):

$$\frac{d\Pi}{dp_t^f} = -w^f + \frac{\partial \pi}{\partial p_t^c}\frac{dp_t^c}{dp_t^f} - T_1(1 - \frac{dp_t^c}{dp_t^f}) - T_2(1 - \frac{dp_t^c}{dp_t^f} - \frac{\partial p_t^a}{\partial p_t^f})(1-\gamma^F) = 0$$
$$\frac{d\Pi}{dp_t^m} = \frac{\partial \pi}{\partial p_t^m} + \frac{\partial \pi}{\partial p_t^c}\frac{dp_t^c}{dp_t^m} - T_1(1 - \frac{dp_t^c}{dp_t^m}) - T_2(1 - \frac{dp_t^c}{dp_t^m} - \frac{\partial p_t^a}{\partial p_t^m})(1-\gamma^F) = 0$$
$$\frac{d\Pi}{da_t} = \frac{\partial \pi}{\partial a_t} + \frac{\partial \pi}{\partial p_t^c}\frac{dp_t^c}{dp_t^a}\frac{\partial p_t^a}{\partial a_t} - T_1(-\frac{dp_t^c}{dp_t^a}\frac{\partial p_t^a}{\partial a_t}) - T_2(\frac{\partial p_t^a}{\partial a_t} - \frac{dp_t^c}{dp_t^a}\frac{\partial p_t^a}{\partial a_t})(1-\gamma^F) = 0$$
$$\frac{d\Pi}{de_t} = \frac{\partial \pi}{\partial e_t} - T_2\frac{\partial p_t^e}{\partial e_t}(1-\gamma^F) = 0$$

Where we have inserted the farmer's estimates $\frac{\partial p_\tau^e}{\partial P_\tau}^F$ and $\frac{\partial P_\tau}{\partial p_t}^F$ instead of $\frac{\partial p_\tau^e}{\partial P_\tau}$ and $\frac{\partial P_\tau}{\partial p_t}$ and used the definition (17) to insert $\gamma^F = \sum_{\tau=t+1}^{\infty} \frac{\partial p_\tau^e}{\partial P_\tau}^F \frac{\partial P_\tau}{\partial p_t}^F (1+\rho^F)^{-\tau}$.

Comparing the first order conditions in (19) with (16), we see that if $\gamma^F = \gamma$ the regulator can implement optimal incentives by setting $T_1 = T_2 = \delta$. Intuitively, if the regulator can measure input and output and he can measure stock changes perfectly, he can calculate emissions in each period indirectly as $\left((p_t^m + p_t^f - p_t^c) - (P_{t+1} - P_t)\right)$. This allows him to implement a 'Pigouvian' emission tax. If farmers have the same estimate of stock emission processes as the regulator, they will perceive the same resulting emission distribution over time as the regulator. If farmers also have the same discount rate as the regulator, then they will perceive those incentives that the regulator intended to induce with the tax. However, it is very optimistic to assume that the regulator can accurately measure phosphorus stocks, that farmers can accurately estimate emission processes and that farmers have a discount rate which is equal to the regulator's. The substantial cost of accurately measuring individual farm's field stocks may mean that the regulator in practice does not have the option of using T_2 as a tax instrument. Further, and perhaps more critically, it seems very unlikely that the individual farmer's subjective discount rates (which include pure time preferences) can ever be equal to the regulator's social discount rate, which should only depend on the expected real growth rate of consumption and the elasticity of marginal utility of consumption. Thus it seems unlikely, that incentives actually perceived by farmers with this emission tax will be the correct incentives.

5. Regulating a farm under asymmetric information – the realistic case

In this section, we develop and investigate the efficiency of a feasible regulatory system for phosphorus emissions from farms when it is not possible to use T_2 as a tax instrument. For this it is useful to differentiate between three different types of farms.

Different types of farms

The limitational production relationship (2) implies that farms can be either phosphorus constrained (where $p_t^c = p_t^m + p_t^f - p_t^a$), or phosphorus *un*constrained (where $p_t^c < p_t^m + p_t^f - p_t^a$).

For all *un*constrained farms, phosphorus fertiliser has no value in crop production. Because phosphorus has no fertilisation value on these farms, they will not use chemical fertiliser, nor will they have any incentive to avoid application emission when applying manure. These are typically farmers who are engaged in intensive livestock production and are so efficient that they find it profitable to produce and apply surplus phosphorus to their fields even though it has no value at the margin for their crops. Since crops are unconstrained and phosphorus is applied in excess, changes in phosphorus application will not affect the crop's uptake of phosphorus (i.e. equation (13) applies). Essentially these farmers have a phosphorus surplus and consider phosphorus in manure to be a waste product which requires disposal at least cost (i.e. dumping it on their fields).

These farmers will continually increase the phosphorus stock in the field over time, as excess applications are incorporated into the stock. Here we distinguish between two types of unconstrained farmers. Type 1 farms are unconstrained farms with $\gamma \approx 1$. These include farms with a phosphorus stock in the field which is either close to, or at the maximum capacity, so that any surplus application is emitted quickly. Type 2 farms are the unconstrained farms where $\gamma < 1$. These farms still have a large remaining phosphorus stock capacity in the soil. Both type 1 and type 2 farms are characterised by surplus application of phosphorous and increasing field stocks (unless they are at maximum capacity).

In contrast, phosphorus constrained farmers (who we call type 3 farms), are in a completely different situation. These are arable farmers, or farms with extensive livestock production, who after manure application still find it profitable to pay a positive price for more phosphorus for their crops in the form of chemical fertiliser or manure. Since $p_t^c = p_t^m + p_t^f - p_t^a$ we have that:

$$\frac{dp_t^c}{dp_t^m} = (1 - \frac{\partial p_t^a}{\partial p_t^m}) \text{ and } \frac{dp_t^c}{dp_t^f} = (1 - \frac{\partial p_t^a}{\partial p_t^f}) \text{ and } \frac{dp_t^c}{dp_t^a} = -1 \quad (20)$$

Intuitively, since crops are phosphorus constrained, all applied phosphorus, which is not lost through application emission, is utilised by the crops. It is not profitable for the farmer to leave phosphorus unused on the field to be incorporated into the soil phosphorus stock. For such farms P_t is not increasing over time (it is actually decreasing because of erosion) and so typically, such farms will have a low phosphorus stock, $P_t << \bar{P}$, and therefore a value of $\gamma \approx 0$ assuming that the regulator discounts the future. Such farmers typically use chemical fertiliser and they find it profitable to reduce phosphorus losses when applying phosphorus to their crops because of the shadow price of phosphorus.

Efficiency of a net application tax

Initially we evaluate efficiency of a simple net application tax by setting $T_2 = 0$ in (19) and comparing the resulting incentives with optimal incentives from (16) for these three types of farms. For type 1 farms (phosphorus unconstrained farms with little or no remaining field stock capacity), we insert (13) and $\gamma = 1$ (into equation 19 and 16). For type 2 farms (phosphorus unconstrained farms with remaining field stock capacity), we insert (13) in both equations, keeping $\gamma < 1$. For type 3 farms (phosphorus constrained farms), we insert (20) into both equations, keeping $\gamma < 1$. The resulting set of first order conditions is presented in table I.

Table I: Efficiency of a net application tax

	Type 1 farms (Phosphorus unconstrained, little or no remaining p-stock capacity) $\gamma \approx 1$		Type 2 farms (Phosphorus unconstrained, substantial remaining p-stock capacity) $\gamma < 1$		Type 3 farms (Phosphorus constrained, substantial remaining p-stock capacity) $\gamma < 1$	
Decision variable	Optimal incentives (16)	Tax Incentives (19)	Optimal incentives (16)	Tax incentives (19)	Optimal incentives (16)	Tax Incentives (19)
p_t^f	δ	T_1	$\delta(\frac{\partial p_t^a}{\partial p_t^f} + (1 - \frac{\partial p_t^a}{\partial p_t^f})\gamma)$	T_1	$\delta \frac{\partial p_t^a}{\partial p_t^f}$	$T_1 \frac{\partial p_t^a}{\partial p_t^f}$
p_t^m	δ	T_1	$\delta(\frac{\partial p_t^a}{\partial p_t^m} + (1 - \frac{\partial p_t^a}{\partial p_t^m})\gamma)$	T_1	$\delta \frac{\partial p_t^a}{\partial p_t^m}$	$T_1 \frac{\partial p_t^a}{\partial p_t^m}$
a_t	0	0	$\delta(1-\gamma)\frac{\partial p_t^a}{\partial a_t}$	0	$\delta \frac{\partial p_t^a}{\partial x_t^a}$	$T_1 \frac{\partial p_t^a}{\partial x_t^a}$
e_t	0	0	$\delta(1-\gamma)\frac{\partial p_t^e}{\partial e_t}$	0	$\delta(1-\gamma)\frac{\partial p_t^e}{\partial e_t}$	0

For unconstrained farms, the tax system's performance depends on the farmer's γ value. For type 1 farms where γ is close to 1, setting $T_1 = \delta$ results in correct marginal incentives for all decision variables (the first two columns). This is not surprising since the emission delay is small when phosphorus is left on the field. Thus, this is almost as if all surplus application of phosphorus is emitted immediately. Since plant uptake is not affected by relaxing the phosphorus constraint, reducing phosphorus losses (from application run off or erosion) has little effect. The resulting emission delay is small because the increase in field stock quickly leads to emissions. The only thing that affects emissions noticeably is reduced manure application and for this the tax system (with $T_1 = \delta$) generates correct incentives.

For unconstrained farms with a small stock (type 2 farms, the middle two columns), incentives are off all across the board. These farms have a large potential for delaying emissions (because of the large remaining phosphorus stock capacity) and so incentives to reduce surplus are 'too large' if the regulator sets $T_1 = \delta$. Setting the correct tax rate for generating the optimal manure application incentives would require the regulator to estimate both γ and $\frac{\partial p_t^a}{\partial p_t^m}$. Furthermore, the tax generates no incentives to reduce application emissions or erosion. This is unfortunate because a reduction in current emissions is added to the field stock, which on these farms results in a substantial (and so valuable) delay in emissions.

Finally, looking at type 3 farms (phosphorus constrained) in the last two columns; setting the tax rate $T_1 = \delta$ gives the correct incentive for manure and fertiliser application, as well as application care. This is off hand surprising since these farms have a large remaining stocking capacity and so the emissions generated by additions to the stock are substantially delayed and thereby causing environmental damage which is much smaller than δ. The reason for this is that these farmers do not stock phosphorus in the fields, but rather they utilise all the applied phosphorus that is not immediately lost to the environment through application run off during the current crop production. Since application does not affect field stocks, and through these future emissions, the dynamic issues that arise when farmers add to stocks through over-application disappear. Only the current period's emissions in connection with application runoff are affected by decisions made during the current period. So, even though γ is small for these farms, setting $T_1 = \delta$ does not distort the incentives for manure/fertiliser application and application care. However, the tax system gives no incentives to invest in erosion reduction (last equation). This may be important because γ is small and so (as for type 2 farms) there are substantial gains from delaying erosion emissions.

Summing up, if the regulator sets $T_1 = \delta$, the net application tax provides the correct incentives for both type 1 and type 3 farms (except that for type 3 farms, supplementing with regulations that create incentives for erosion reduction measures are needed). For type 2 farms, on the other hand, information about each individual farm is required for the regulator to be able to set the 'correct' tax rate for the net application tax. However, even if this is done and the needed supplementing regulations that create incentives for erosion reduction measures are implemented, this still does not give incentives to reduce losses from application run off.

6. Regulating a heterogeneous farm sector in practice

In the previous sections, we investigated the optimality of incentives generated by a feasible net application tax for different types of farms when the tax rate was set equal to marginal environmental damage. In this section, we address the implementation of such a tax for the regulation of phosphorus emissions from a heterogeneous farm sector, initially consisting of type 1, 2 and 3 farms. Clearly, if the cost of transporting manure was negligible, type 1 and 2 farms, with a shadow value of phosphorus less than w^f, would be able to export manure to type 3 farms which have a shadow value of phosphorus equal to w^f. Probably the most important reason why all three types of farms can coexist (as we see in many countries), is that the cost of manure transportation is high, so that trade which increase welfare are only possible within a small radius around type 1 and 2 farms. If the concentration of intensive livestock farms in a locality is high, the demand for phosphorous from type 3 farms within this radius may be insufficient to eliminate excessive application and some intensive livestock farms will remain unconstrained (type 1 and 2).

In this situation, a regulator should ideally apply different tax rates to the different types of farms. However, the possibility for differentiating tax rates between farms is limited as, e.g. high protein feed is easy to transport and trade ‘illegally’ between farms. For example, if the net application tax was applied with different rates for different farms, it would be very difficult to prevent type 2 farms, with a low tax rate, from importing high protein feed and selling it illegally to farms with a high tax rate[12]. In practice, a net application tax would have to be applied with a uniform rate for the entire sector (or only differentiated between areas where effective border control can be implemented). Thus, the

[12] This problem has been pointed out in the literature (see e.g. Hansen, 1999). Fraud has occurred in the Dutch MINAS system (Oenema and Berentsen 2005).

policy design problem is to find the (second best) optimal uniform tax rate that can be applied to all farms in the industry and to find feasible supplementary regulations that can correct important remaining deviations from the optimal incentives. In some cases, the need for differentiation may be quite small. When the regulator implements a uniform net application tax, it not only has effects on farm incentives, but it also increases incentives to trade manure between farms. Such a tax will increase the shadow value of phosphorous on type 3 farms by the tax rate, while the shadow value of phosphorous on the remaining type 1 and 2 farms with over application will still be zero. This increases the radius around the remaining type 2 and 3 farms within which manure trading is economically advantageous. Thus, more manure from surplus farms (type 1 and type 2) will be exported to constrained farms (type 3). In localities where the density of intensive livestock farms is not too great, the result may well be that all phosphorous surplus from type 1 and type 2 farms is sold to type 3 farms. In this case, all type 1 and type 2 farms in the locality become phosphorus constrained and in effect become type 3 farms (with a shadow price of phosphorus equal to the local market price net of transport costs). Since the net application tax implements optimal application incentives for type 3 farms when $T_1 = \delta$, incentives to trade and reallocate manure are also optimal. Thus, if the proportion of type 2 and type 3 farms remaining in the agricultural sector *after* implementation of the tax is small, the optimal uniform tax rate will be close to δ. In this case, the net application tax will generate close to optimal incentives on most farms (see table I) with one exception. The former type 2 farms with a γ parameter which is substantially lower than 1, but greater than zero, should be given an incentive to reduce erosion losses since these incentives are not generated by the tax.

One way to generate this incentive could be to supplement the tax with a subsidy for verifiable erosion reduction measures. Only farms with the relevant γ parameter size should be eligible for a subsidy and this parameter could be estimated for any farm applying for the subsidy. Since eligible farmers are subsidised, they have an incentive to apply for and document subsidised activities, so that the regulator only needs to verify that these activities have taken place. It might be feasible for regulators to verify some effective erosion reduction measures, such as increasing the width of uncultivated buffers between fields and surface waters, the establishment of wetland areas and winter tillage. Thus, supplementing the general net application tax with such a subsidy scheme that gives relevant farms incentives to reduce erosion, could result in close to optimal abatement incentives (in the sense described in the previous section – see table I) for most farms in the sector.

However, if there are localities where the density of type 1 and type 2 farms is so large that, even when all possibilities for substituting with chemical fertiliser on type 3 farms subsequent to the implementation of the tax have been exploited, surplus application still occurs on some farms that therefore remain as type 1 and type 2 farms. In such localities, supplementing the tax with a subsidy for verifiable erosion reduction measures on relevant farms will not be enough to ensure close to optimal incentives. As indicated in table I above, two important incentive problems remain:

- Type 2 farms are not given incentives to reduce application loss, and
- Type 1 farms are not given incentives to reallocate surplus application of manure to type 2 farms

Addressing the first distortion is difficult, since reducing application loss is mainly a question of increasing the farmers' care and effort in connection with application. Rules about the timing and weather conditions under which application is allowed attempt to address this, but effectively controlling such regulations is difficult.

The second distortion may be important in such localities because type 1 farms with surplus manure continue to apply it to their saturated fields resulting in the rapid loss of applied excess phosphorus to the environment. If the over-application of phosphorus was shifted to a neighbouring type 2 farm this would not increase utilisation of manure phosphorus in plant production. However, if this type 2 farm has a γ value substantially lower than 1, then shifting over application to this farm would substantially delay the resulting loss to the environment and generate an increase in welfare. The problem is that the uniform net application tax does not generate the (optimal) incentives for this transfer. The optimal incentives and the incentives generated by the net application tax are presented in table II, where farmer *i* is the type 1 farmer considering selling manure to his type 2 neighbour. :

Table II: Incentives in optimum under the net application tax

	Farmer *i* (type 1 optimally exporting manure)		Farmer *j* (type 2 optimally importing manure)	
Decision variable	Optimal incentives	Tax incentives	Optimal incentives	Tax Incentives
p_t^m	δ	T_1	$\delta(\frac{\partial p_j^a}{\partial p_j^m}+(1-\frac{\partial p_j^a}{\partial p_j^m})\gamma_j)$	T_1

Under optimal incentives difference in the tax rates paid by the two types of farmers correspond to the difference in their γ values accurately reflecting the difference in the discounted environmental damage that results from surplus application of phosphorus. However, because the net application tax does not provide optimal differentiated application incentives for type 2 farmers, there is no incentive for trading manure between unconstrained farmers. Since these areas also have the highest emissions of phosphorus, exploiting this potential for delaying the emissions may be important.

Supplementing the net applications tax with a tax rebate scheme

A simple and feasible way to exploit some of this potential for reallocating excess manure application could be through a scheme of tax rebates. The regulator could allocate a quota of free, or reduced fee, phosphorus surplus application to qualified farms. Such a quota should only be given to selected farms with small γ in intensive livestock areas where there is a general surplus application problem. When a farmer applies for a quota, the regulator inspects the farm and estimates its erosion emission function (5), current phosphorus stock in the fields and γ given current production practices. Based on this, a quota for surplus application at a reduced fee could be allocated. The criteria should be that adding this quota to the field stock does not substantially increase γ. Allocations should only be given to farms with small γ and use of the quota should be conditional on the verified application of manure from farms with a certified high γ, i.e. these farms must also be inspected and approved and their γ estimated. The rebate should depend on the difference between the estimated verified γs, i.e. so that the verified application of q kilos of phosphorus from a certified high γ farm gives a refund $q\delta$ times the difference between the two farms' γ. For the farms accepted into the scheme, the rebate would generate an incentive to shift surplus manure application between farms that reflect the environmental benefit of doing so (i.e. coming closer to the optimal reallocation incentives in table II).

Such a scheme would be administratively costly, but might be feasible to implement and control. In the quota system, farms are allocated an over-application quota based on individual estimates of phosphorus input and output and on phosphorus emission probabilities. These estimates are made when farmers apply for the program and could be based on the same type of soil tests and inspections that are currently used for the calculation of the farm's phosphorus index[13]. For this scheme to work, it is critical that regulators can verify that supplies of manure for which a tax rebate has been claimed have actually been delivered and are not just "on paper deliveries". Since the reallocated manure has no fertilisation value for either farmer involved in the transfer, both have an interest in saving transportation costs if it was possible to claim the rebate without actually transferring manure. The advantage here is that, since such deliveries imply a tax rebate, they only need to be verifiable. Thus, rebate payments could be made conditional on deliveries being made at specified times, reported in advance and subject to random inspection, or even that deliveries must be made by certified third party transporters. The payment of rebates for the entire allocated quota could also be made on the condition that the farmer is never caught cheating during random inspections.

[13]The phosphorus index is an indicator of the risk of phosphorus being lost from a specific field to the nearest water body based on, among other things, the current field stock of phosphorous, estimated amounts of phosphorus applied and technology parameters such as drainage, slope of the field, buffer zones and distance from the damaged ecosystem. In the US, the phosphorus index is used to point out areas for phosphorus regulation (Andersen and Kronvang 2006, Sharpley et al. 2003).

7. Concluding Discussion

In this paper, we develop a model of phosphorus loss from agriculture and its regulation. The model incorporates both the important stock dynamics which characterise phosphorus application and loss, and important feasibility constraints on regulatory alternatives due to problems of information asymmetry and enforcement. The analysis highlights an enlightening typology of farms, which basically distinguishes between phosphorus constrained farms, on which phosphorus is a valuable nutrient with a positive shadow price, and unconstrained farms, on which phosphorus is a waste product with no shadow value. It is the latter type of farm that is the main cause of current emission problems in many countries. The reason that constrained and unconstrained farms can coexist is that manure transportation costs limit the radius for profitable trading of manure between such farms. This also implies that an important reason why intensive livestock farms become unconstrained is that they are located in areas where the demand for manure from constrained farms at current prices is ‘too small’ (e.g. because the concentration of intensive livestock farms in the area is high).

We conclude that a uniform net application tax could be a feasible core regulation in a system of regulations generally generating close to optimal incentives. In addition to on farm incentives, the tax increases the radius within which trade between constrained and unconstrained farms is profitable, thereby reducing the number of unconstrained ‘problem’ farms. If the proportion of farms in the sector which remain unconstrained *after* implementation of the tax is small, supplementing the tax with a subsidy for verifiable erosion reduction measures would generate close to optimal incentives for most farms.

If many farms remain unconstrained after implementation of the tax, two more incentive distortions remain unsolved by the tax. We suggest supplementing the tax with a tax rebate scheme to induce a shift of excess manure application from farms with saturated fields to farms with unsaturated fields. This would delay the loss of phosphorus to the environment, which would otherwise happen quickly if the excess application continued on the saturated fields. This scheme therefore addresses one of the remaining incentive distortions. However, the other remaining distortion, that unconstrained farmers are not given an incentive to reduce application loss, is difficult to address.

Though the supplemented tax scheme in this case leaves important short run distortions unsolved, there is an additional long run argument for tax regulation. An important reason why intensive livestock farms remain unconstrained after implementation of the tax is that they are located in areas where the demand for manure from constrained farms is limited because the concentration of intensive livestock farms in the area is high. Because of sunk capital costs, it is costly to move livestock production to other locations. However, providing farmers with the correct incentives when they decide to investment in intensive livestock farm capital is important. The net application tax would ensure this by providing farmers with an incentive to localise intensive livestock farms in areas where there is ample demand for manure from constrained farmers. Farmers comparing different localities for investment will be comparing localities where the market price for surplus manure is zero with other localities where the price can be as high as fertiliser costs plus the tax rate. Since unconstrained intensive livestock farms are the cause of current environmental problems, the correct localisation of intensive livestock farms is probably of primary importance in the long run.

The most important qualification to our analysis of phosphorus loss and the conclusions we draw about how to regulate it, is that throughout, we assume that environmental damage does not vary between farms. This is often an unrealistic assumption. If the variation in damage between farms and localities is substantial, it is important that the incentives generated by regulation reflect this variation. Because of the problems of controlling 'illegal' trade in, e.g. high protein feed between farms, the net application tax suggested here cannot be differentiated. In this case, the suggested tax and the supplementary schemes could be given the role of a general cost-effective baseline regulation, which generates incentives reflecting the damage caused by phosphorus emissions in low damage areas. In localities where marginal damage from phosphorous pollution is high, the baseline regulation system would have to be supplemented by additional regulations aimed at immobile and observable behaviour or investments that make it possible for the regulator to control compliance effectively. Since these supplementary regulations are imprecise and costly to control in high damage areas, it is probably efficient to increase the intensity of the baseline incentives so as to reduce the extent of the necessary supplementary regulation in high damage areas. However, if damage from phosphorus emissions is negligible for a substantial part of the agricultural sector, this could completely undermine the argument for tax regulation. In this case, it would be more efficient only to implement regulations aimed at immobile and observable behaviour in the few areas where emissions cause damage and leave the remaining part of the farm sector unregulated.

Finally, we should stress that our analysis assumes that correct regulation of other externalities from farm production is in place. This may be particularly important for the environmental problems caused by nitrogen emissions from farm fields because they are especially entangled with the phosphorous emissions problem as both nutrients are imported to fields through manure application. Thus, an important qualification to our conclusions could result if the proper regulation of nitrogen emissions is not already in place (or if the imposed regulations imply inefficient abatement incentives for nitrogen). If existing inefficient nitrogen regulations cannot be improved for some reason, several researchers (e.g. Weersink et al. 2004, Helin et al. 2006) have suggested that this could be taken into account when designing phosphorous regulations.

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Appendix A: The Phosphorus problem in Denmark and the US

During the past few decades, a number of countries have seen an increase in the number and size of intensive livestock farms (in Denmark typically pig farms, in the US, cattle and poultry farms) which are characterised by substantial surplus nutrient application. Phosphorus leaching from these farms can be mediated for some time because of the natural stocking capacity of phosphorus in the soil, but a number of these farms have now reached the limit of their fields' stocking capacity for phosphorus[14]. Today, these farms are the cause of substantial emissions of phosphorus to the environment and more of these intensive livestock production farms will, in the future, reach their stocking capacity if the over-application of phosphorus continues. In Denmark, a number of soils are tested every year to determine the content of different nutrients. The phosphorus content is determined by the soil test P (Olsen P method), which gives the amount of plant available phosphorus in the soil (mg P/100 g). Soils which have phosphorus content lower than 2 are considered to be in phosphorus deficit. Soils which have a phosphorus number between 2 and 4 are optimally fertilised, whilst soils with a phosphorus number above 4 contain excessive phosphorus. If the phosphorus number is 6 or above, the soil is characterised as being saturated, or as having exceeded its stocking capacity. Table A1 presents the distribution of the tested Danish soils for the years 1997, 2001 and 2010 together with the Danish Agriculture and Food Council's fertilisation recommendations.

Table A1: Phosphorus numbers in Danish soils, per cent distribution

	1997	2000	2010	Recommendations

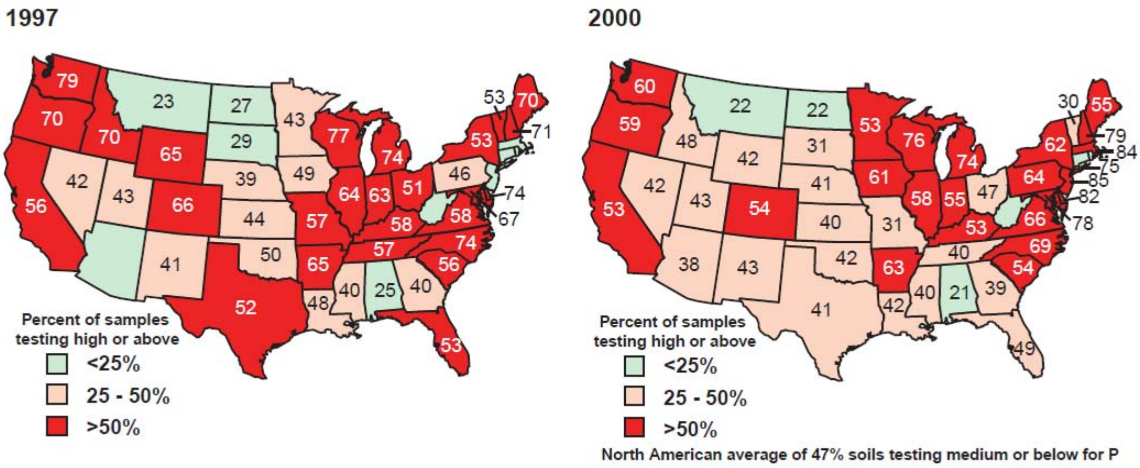
[14]As noted earlier, during recent decades, phosphorus application to fields has been constrained by other regulations which are typically those aimed at nitrogen reduction. However, constraints on, e.g. the application of manure designed to ensure 'reasonable' levels of nitrogen application, typically make the substantial over-application of phosphorus possible (Sharpley et al. 2003).

<1	1	1	1	Application of large amounts of P to increase the P number
1-2	6	8	7	Application of 20-40 % more P than removed by plants
2-4	48	52	51	Application of P corresponding to average plant uptake
4-6	35	31	31	Application of only 25-50 % of plant uptake
>6	11	9	10	In the short run, no application of P
No. of soil analysis	87,439	54,863	71,253	

Source: Rubæk et al. (2005), Pedersen (1997, 2000), Pedersen (2010).

We see from table A1 that the proportion of analysed soils which are close to their maximum capacity, or have already attained that status for all three years, is about 40 %. This is despite the extensive regulation of nitrogen (and secondary phosphorus) use in Danish agriculture. A low percentage of the soils are in deficit, whilst about half are in the range of the optimum phosphorus content according to plant production. Figure A1 presents two maps from 1997 and 2000, respectively, which show soils in the US with the percentage of soil samples testing high or above in the soil test P (Mehlich-1 method) analysis near phosphorus sensitive waters.

Figure A1: Per cent of soil samples testing high or above in the soil test P analysis in the US



Source: Sharpley et al. 2003.

Figure A1 shows how many soil samples tested high or above for phosphorous in the soil test P and thus require little or no fertilisation. The figure shows that high phosphorus levels are a regional problem which is associated with intensive livestock production, typically poultry or cattle production (Sharpley et al. 2003).

Appendix B: Mathematical appendix: solving the social welfare maximisation problem

The social welfare maximisation problem is defined in equation (14):

$$\underset{p_t^m, x_t^a, x_t^e, p_t^f, p_t^c}{MAX} SW = \sum_{t=0}^{\infty} \left[\pi((p_t^m, p_t^c, x_t^a, x_t^e) - w^f p_t^f) - \delta(p_t^a + p_t^e) \right] (1+\rho)^{-t} \qquad (14)$$

Subject to:

$$p_t^a = p_t^a(p_t^m, p_t^f, a_t) \qquad (4)$$

$$p_t^c \leq p_t^m + p_t^f - p_t^a \qquad (2)$$

$$p_t^e = p_t^e(P_t, e_t) \qquad (5)$$

$$p_t = p_t^m + p_t^f - p_t^c - p_t^a - p_t^e \qquad (6)$$

$$P_t = P_0 + \sum_{\tau=0}^{t-1} p_\tau \qquad \text{for } t=1....\infty \qquad (8)$$

Inserting all the above equations into (14) gives the maximisation problem (A):

$$\underset{p_t^m, x_t^a, x_t^e, p_t^f, p_t^c}{MAX} SW = \sum_{t=0}^{\infty} \begin{bmatrix} \pi((p_t^m, \min(p_t^c, p_t^m + p_t^f - p_t^a(p_t^m, p_t^f, x_t^a)), x_t^a, x_t^e) - w^f p_t^f) - \\ \delta(p_t^a(p_t^m, p_t^f, x_t^a) + p_t^e((P_0 + \sum_{\tau=0}^{t-1} p_t^m + p_t^f - p_t^c - p_t^a - p_t^e), x_t^e)) \end{bmatrix} (1+\rho)^{-t}$$

Differentiate (A) with respect to p_t^m and using equation (15):

$$0 \leq \sum_{\tau=t+1}^{T} \frac{\partial p_\tau^e}{\partial P_\tau} \frac{\partial P_\tau}{\partial p_t} (1+\rho)^{-t} \rightarrow \gamma \leq 1 for T \rightarrow \infty \qquad (15)$$

We get:

$$\frac{\partial SW}{\partial p_t^m} = 0 = \frac{\partial \pi}{\partial p_t^m} + \frac{\partial \pi}{\partial p_t^c}\frac{dp_t^c}{dp_t^m} - \delta\frac{\partial p_t^a}{\partial p_t^m} + \delta(\frac{\partial p_t^m}{\partial p_t^m}\gamma - \frac{dp_t^c}{dp_t^m}\gamma - \frac{\partial p_t^a}{\partial p_t^m}\gamma) =$$
$$\frac{\partial \pi}{\partial p_t^m} + \frac{\partial \pi}{\partial p_t^c}\frac{dp_t^c}{dp_t^m} - \delta(\frac{\partial p_t^a}{\partial p_t^m} + (1 - \frac{\partial p_t^c}{\partial p_t^m} - \frac{\partial p_t^a}{\partial p_t^m})\gamma)$$

In the same way as above, changes due to p_t^f, x_t^a *and* x_t^e are calculated.