Appendix C: Infinite rotations

1. Relaxing some of the basic assumptions

1.1. Incorporating pollution

1.1.1. A private optimum

Here the aquaculture producer maximizes:

$$Max[\frac{P(T,q(T),s_t)}{e^{rT}-1}] \tag{C.1}$$

With T and s_t as control variables, the first-order conditions are:

$$\frac{[\frac{\partial P}{\partial T}+\frac{\partial P}{\partial q}\frac{\partial q}{\partial T}](e^{rT}-1)-re^{rT}P(T,q(T),s_t)}{(e^{rT}-1)^2}=0 \tag{C.2}$$

$$\frac{\frac{\partial P}{\partial s_t}}{e^{rT}-1}=0 \tag{C.3}$$

(C.2) and (C.3) can be reduced to:

$$\frac{\partial P}{\partial T}+\frac{\partial P}{\partial q}\frac{\partial q}{\partial T}-\frac{re^{rT}}{e^{rT}-1}P(T,q(T),s_t)=0 \tag{C.4}$$

$$\frac{\partial P}{\partial s_t}=0 \tag{C.5}$$

To find the private optimal pollution level, $\bar{s}$, and rotation time, T_p*, we need to solve (C.4) and (C.5) as two equations with two unknowns.

1.1.2. A social optimum

The social planner maximizes:

$$Max[\frac{P(T,q(T),s_t)-D(s_t)}{e^{rT}-1}] \tag{C.6}$$

The first-order conditions are:

$$\frac{[\frac{\partial P}{\partial T}+\frac{\partial P}{\partial q}\frac{\partial q}{\partial T}](e^{rT}-1)-re^{rT}(P(T,q(T),s_t)-D(s_t))}{(e^{rT}-1)^2}=0 \tag{C.7}$$

$$\frac{\frac{\partial P}{\partial s_t}-D'(s_t)}{e^{rT}-1}=0 \tag{C.8}$$

(C.7) and (C.8) can be reduced to:

$$\frac{\partial P}{\partial T}+\frac{\partial P}{\partial q}\frac{\partial q}{\partial T}-\frac{re^{rT}}{e^{rT}-1}P(T,q(T),s_t)+\frac{re^{rT}}{e^{rT}-1}D(s_t)=0 \qquad \text{(C.9)}$$

$$\frac{\partial P}{\partial s_t}-D'(s_t)=0 \qquad \text{(C.10)}$$

To find the social optimal pollution level, s_t*, and rotation time, , T_s*, we have two solve (C.9) and (C.10) as two equations with two unknowns. Furthermore, when comparing (C.4) with (C.9) we see that $T_s* > T_p*$, while $s_t* < \bar{s}$ based on (C.5) and (C.10).

1.1.3. Optimal regulation

Since $T_s* > T_p*$, we need a subsidy to increase the private rotation time. By comparing (C.4) and (C.9) we get that:

$$\frac{\partial U}{\partial T}=-\frac{rD(s_t)}{1-e^{-rT}} \qquad \text{(C.11)}$$

Comparing (C.5) and (C.10) we see that the following marginal tax on pollution provides an optimum:

$$\frac{\partial V}{\partial s_t}=D'(s_t*) \qquad \text{(C.12)}$$

(C.12) is exactly a Pigovian tax.

1.2. Fixed costs

1.2.1. A private optimum

Now the aquaculture producer maximizes:

$$Max[\frac{P(T,q(T))-FP}{e^{rT}-1}] \qquad \text{(C.13)}$$

With T as control variable, the first-order condition is:

$$\frac{[\frac{\partial P}{\partial T}+\frac{\partial P}{\partial q}\frac{\partial q}{\partial T}](e^{rT}-1)-re^{rT}(P(T,q(T))-FP)}{(e^{rT}-1)^2}=0 \qquad \text{(C.14)}$$

(C.14) reduces to:

$$\frac{\partial P}{\partial T}+\frac{\partial P}{\partial q}\frac{\partial q}{\partial T}-\frac{re^{rT}}{e^{rT}-1}P(T,q(T))+\frac{re^{rT}}{e^{rT}-1}FP=0 \qquad \text{(C.15)}$$

1.2.2. A social optimum

The social planner maximizes:

$$Max[\frac{P(T,q(T))-\int_0^T D(x)dx-FS}{e^{rT}-1}] \tag{C.16}$$

Using *T* as control variable, the first-order condition is:

$$\frac{[\frac{\partial P}{\partial T}+\frac{\partial P}{\partial q}\frac{\partial q}{\partial T}-D(T)](e^{rT}-1)}{(e^{rT}-1)^2}-$$

$$\frac{re^{rT}(P(T,q(T))-\int_0^T D(x)dx-FS)}{(e^{rT}-1)^2} \tag{C.17}$$

(C.17) can be reduced to:

$$\frac{\partial P}{\partial T}+\frac{\partial P}{\partial q}\frac{\partial q}{\partial T}-D(T)-\frac{re^{rT}}{e^{rT}-1}P(T,q(T))+$$

$$\frac{re^{rT}}{e^{rT}-1}\int_0^T D(x)dx+\frac{re^{rT}}{e^{rT}-1}FS=0 \tag{C.18}$$

1.2.3. Optimal regulation

By comparing (C.15) and (C.18) and using the same procedure as in appendix B, we get the following marginal tax:

$$\frac{\partial U}{\partial T}=D(T)-\frac{r\int_0^T D(x)dx}{1-e^{-rT}}-\frac{r(FS-FP)}{1-e^{-rT}} \tag{C.19}$$

1.3. Running costs

1.3.1. A private optimum

Here the problem of the aquaculture producer becomes:

$$Max[\frac{R(T,q(T))-\int_0^T CP(y)dy}{e^{rT}-1}] \tag{C.20}$$

When using *T* as control variable, we obtain the following the first-order condition:

$$\frac{[\frac{\partial R}{\partial T}+\frac{\partial R}{\partial q}\frac{\partial q}{\partial T}-CP(T)](e^{rT}-1)}{(e^{rT}-1)^2}-$$

$$\frac{re^{rT}(R(T,q(T))-\int_0^T CP(y)dy)}{(e^{rT}-1)^2}=0 \tag{C.21}$$

(C.21) reduces to:

$$\frac{\partial R}{\partial T}+\frac{\partial R}{\partial q}\frac{\partial q}{\partial T}-CP(T)-$$

$$\frac{re^{rT}}{e^{rT}-1}R(T,q(T))+\frac{re^{rT}}{e^{rT}-1}\int_0^T CP(y)dy=0 \tag{C.22}$$

1.3.2. A social optimum

The social planner maximizes:

$$Max[\frac{R(T,q(T))-\int_0^T CS(y)dy-\int_0^T D(x)dx}{e^{rT}-1}] \tag{C.23}$$

The first-order condition is:

$$\frac{[\frac{\partial R}{\partial T}+\frac{\partial R}{\partial q}\frac{\partial q}{\partial T}-CS(T)-D(T)](e^{rT}-1)}{(e^{rT}-1)^2}-$$

$$\frac{re^{rT}(R(T,q(T))-\int_0^T CS(y)dy-\int_0^T D(x)dx)}{(e^{rT}-1)^2} \tag{C.24}$$

(C.24) can be reduced to:

$$\frac{\partial R}{\partial T}+\frac{\partial R}{\partial q}\frac{\partial q}{\partial T}-CS(T)-D(T)-\frac{re^{rT}}{e^{rT}-1}R(T,q(T)+$$

$$\frac{re^{rT}}{e^{rT}-1}\int_0^T D(x)dx+\frac{re^{rT}}{e^{rT}-1}\int_0^T CS(y)dy=0 \tag{C.25}$$

1.3.3. Optimal regulation

By comparing (C.22) and (C.25) we get the following marginal tax:

$$\frac{\partial U}{\partial T} = D(T) + (CS(T) - CP(T)) - \frac{r\int_0^T D(x)dx}{1-e^{-rT}} - \frac{r(\int_0^T CS(y)dy - \int_0^T CP(y)dy)}{1-e^{-rT}} \tag{C.26}$$

1.4. Another formulation of quality

1.4.1. A private optimum

Now the aquaculture producer's objective function is:

$$Max[\frac{RP(T) - C(T)}{e^{rT} - 1}] \tag{C.27}$$

With T as control variable, the first-order condition is:

$$\frac{[RP'(T) - C'(T)](e^{rT} - 1) - re^{rT}(RP(T) - C(T))}{(e^{rT} - 1)^2} = 0 \tag{C.28}$$

(C.28) reduces to:

$$RP'(T) - C'(T) - \frac{re^{rT}}{e^{rT} - 1} RP(T) + \frac{re^{rT}}{e^{rT} - 1} C(T) = 0 \tag{C.29}$$

1.4.2. A social optimum

The social planner's maximization problem is:

$$Max[\frac{RS(T) - C(T) - \int_0^T D(x)dx}{e^{rT} - 1}] \tag{C.30}$$

By using T as control variable, the first-order condition is:

$$\frac{[RS'(T) - C'(T) - D(T)](e^{rT} - 1)}{(e^{rT} - 1)^2} - \frac{re^{rT}(RS(T) - C(T) - \int_0^T D(x)dx)}{(e^{rT} - 1)^2} = 0 \tag{C.31}$$

(C.31) can be reduced to:

$$RS'(T) - C'(T) - D(T) - \frac{re^{rT}}{e^{rT}-1}RS(T) + \frac{re^{rT}}{e^{rT}-1}C(T) + \frac{re^{rT}}{e^{rT}-1}\int_0^T D(x)dx = 0 \tag{C.32}$$

1.4.3. Optimal regulation

By comparing (C.29) and (C.32) we get the following marginal tax:

$$\frac{\partial U}{\partial T} = D(T) - \frac{r\int_0^T D(x)dx}{1-e^{-rT}} + \frac{r(RS(T)-RP(T))}{1-e^{-rT}} - RS'(T) + RP'(T) \tag{C.33}$$

1.5. Private and social discount rates

1.5.1. A private optimum

Now the aquaculture producer's optimization problem is:

$$Max[\frac{P(T,q(T))}{e^{r_pT}-1}] \tag{C.34}$$

With T as control variable, the first-order condition is:

$$\frac{[\frac{\partial P}{\partial T} + \frac{\partial P}{\partial q}\frac{\partial q}{\partial T}](e^{r_pT}-1) - r_p e^{r_pT}P(T,q(T))}{(e^{r_pT}-1)^2} = 0 \tag{C.35}$$

(C.35) reduces to:

$$\frac{\partial P}{\partial T} + \frac{\partial P}{\partial q}\frac{\partial q}{\partial T} - \frac{r_p e^{r_pT}}{e^{r_pT}-1}P(T,q(T)) = 0 \tag{C.36}$$

1.5.2. A social optimum

The social planner maximizes:

$$Max[\frac{P(T,q(T)) - \int_0^T D(x)dx}{e^{r_sT}-1}] \tag{C.37}$$

The social planner's first-order condition is:

$$\frac{[\frac{\partial P}{\partial T} + \frac{\partial P}{\partial q}\frac{\partial q}{\partial T} - D(T)](e^{r_sT}-1)}{(e^{r_sT}-1)^2} -$$

$$\frac{r_s e^{r_s T}(P(T,q(T)) - \int_0^T D(x)dx)}{(e^{r_s T}-1)^2} \tag{C.38}$$

(C.38) can be reduced to:

$$\frac{\partial P}{\partial T} + \frac{\partial P}{\partial q}\frac{\partial q}{\partial T} - D(T) - \frac{r_s e^{r_s T}}{e^{r_s T}-1}P(T,q(T)) +$$

$$\frac{r_s e^{r_s T}}{e^{r_s T}-1}\int_0^T D(x)dx = 0 \tag{C.39}$$

1.5.3. Optimal regulation

By comparing (C.36) and (C.39) we get the following marginal tax:

$$\frac{\partial U}{\partial T} = D(T) - \frac{r\int_0^T D(x)dx}{1-e^{-rT}} + (\frac{r_s}{1-e^{-r_s T}} - \frac{r_p}{1-e^{r_p T}})P(T,q(T)) \tag{C.40}$$