Appendix B: Basic models

1. A private optimum

The aquaculture producer maximizes:

$$Max[\frac{P(T,q(T))}{e^{rT}-1}] \tag{B.1}$$

When using T as control variable, the first-order condition is:

$$\frac{[\frac{\partial P}{\partial T}+\frac{\partial P}{\partial q}\frac{\partial q}{\partial T}](e^{rT}-1)-re^{rT}P(T,q(T))}{(e^{rT}-1)^2}=0 \tag{B.2}$$

By reducing $e^{rT}-1$ away, (B.2) can be written as:

$$\frac{[\frac{\partial P}{\partial T}+\frac{\partial P}{\partial q}\frac{\partial q}{\partial T}](e^{rT}-1)-re^{rT}P(T,q(T))}{e^{rT}-1}=0 \tag{B.3}$$

Applying the denominator separately to each component of (B.3) yields:

$$\frac{\partial P}{\partial T}+\frac{\partial P}{\partial q}\frac{\partial q}{\partial T}-\frac{re^{rT}}{e^{rT}-1}P(T,q(T))=0 \tag{B.4}$$

Reorganizing (B.4) gives:

$$\frac{\partial P}{\partial T}=\frac{re^{rT}}{e^{rT}-1}P(T,q(T))-\frac{\partial P}{\partial q}\frac{\partial q}{\partial T} \tag{B.5}$$

By dividing with $P(T,q(T))$ and reorganizing, (B.5) can be written as:

$$\frac{\frac{\partial P}{\partial T}}{P(T,q(T))}=r(\frac{e^{rT}}{e^{rT}-1})-\frac{\frac{\partial P}{\partial q}\frac{\partial q}{\partial T}}{P(T,q(T))} \tag{B.6}$$

Now $\frac{e^{rT}}{e^{rT}-1}=\frac{\frac{e^{rT}}{e^{rT}}}{\frac{e^{rT}}{e^{rT}}-\frac{1}{e^{rT}}}=\frac{1}{1-e^{-rT}}$, and by using this fact in (B.6), we get that:

$$\frac{\frac{\partial P}{\partial T}}{P(T,q(T))}=r(\frac{1}{1-e^{-rT}})-\frac{\frac{\partial P}{\partial q}\frac{\partial q}{\partial T}}{P(T,q(T))} \tag{B.7}$$

(B.7) is identical to (4) in the main text.

2. A social optimum

The social planner maximizes:

$$Max[\frac{P(T,q(T))-\int_0^T D(x)dx}{e^{rT}-1}] \tag{B.8}$$

When using T as control variable, the first-order condition is:

$$\frac{[\frac{\partial P}{\partial T}+\frac{\partial P}{\partial q}\frac{\partial q}{\partial T}-D(T)](e^{rT}-1)-re^{rT}(P(T,q(T))-\int_0^T D(x)dx)}{(e^{rT}-1)^2}=0 \tag{B.9}$$

By reducing $e^{rT}-1$, away, (B.9) can be written as:

$$\frac{[\frac{\partial P}{\partial T}+\frac{\partial P}{\partial q}\frac{\partial q}{\partial T}-D(T)](e^{rT}-1)-re^{rT}(P(T,q(T))-\int_0^T D(x)dx)}{e^{rT}-1}=0 \tag{B.10}$$

Applying the denominator separately to each component of (B.10) yields:

$$\frac{\partial P}{\partial T}+\frac{\partial P}{\partial q}\frac{\partial q}{\partial T}-D(T)-\frac{re^{rT}}{e^{rT}-1}P(T,q(T))+\frac{re^{rT}}{e^{rT}-1}\int_0^T D(x)dx=0 \tag{B.11}$$

Reorganizing (B.11) gives:

$$\frac{\partial P}{\partial T}=\frac{re^{rT}}{e^{rT}-1}P(T,q(T))-\frac{re^{rT}}{e^{rT}-1}\int_0^T D(x)dx-\frac{\partial P}{\partial q}\frac{\partial q}{\partial T}+D(T) \tag{B.12}$$

By dividing with $P(T,q(T))$ and reorganizing, (B.5) can be written as:

$$\frac{\frac{\partial P}{\partial T}}{P(T,q(T))}=r(\frac{e^{rT}}{e^{rT}-1}-\frac{e^{rT}}{e^{rT}-1}\frac{\int_0^T D(x)dx)}{P(T,q(T))})-\frac{\frac{\partial P}{\partial q}\frac{\partial q}{\partial T}}{P(T,q(T))}+\frac{D(T)}{P(T,q(T))} \tag{B.13}$$

Now, $\frac{e^{rT}}{e^{rT}-1}=\frac{\frac{e^{rT}}{e^{rT}}}{\frac{e^{rT}}{e^{rT}}-\frac{1}{e^{rT}}}=\frac{1}{1-e^{-rT}}$, and by using this fact in (B.13), we get that:

$$\frac{\frac{\partial P}{\partial T}}{P(T,q(T))} = r\left(\frac{1}{1-e^{-rT}} - \frac{\int_0^T D(x)dx}{P(T,q(T))(1-e^{-rT})}\right) - \frac{\frac{\partial P}{\partial q}\frac{\partial q}{\partial T}}{P(T,q(T))} + \frac{D(T)}{P(T,q(T))} \tag{B.14}$$

(B.14) is identical to (8) in the main text.

3. Optimal regulation

By comparing (B.4) and (B.11), we get the following marginal tax imply that $T_P^* = T_S^*$:

$$\frac{\partial U}{\partial T} = D(T) - \frac{re^{rT}}{e^{rT}-1}\int_0^T D(x)dx \tag{B.15}$$

By using the fact that $\frac{e^{rT}}{e^{rT}-1} = \frac{\frac{e^{rT}}{e^{rT}}}{\frac{e^{rT}}{e^{rT}} - \frac{1}{e^{rT}}} = \frac{1}{1-e^{-rT}}$, (B.15) can be written as:

$$\frac{\partial U}{\partial T} = D(T) - \frac{r\int_0^T D(x)dx)}{1-e^{-rT}} \tag{B.16}$$

(B.16) is identical to (9) in the main text.