Appendix A: A single rotation

1. Basic model

1.1. A private optimum

The problem of the aquaculture producer can be written as:

$$Max[P(T,q(T))e^{-rT}] \tag{A.1}$$

With T as control variable, the first-order condition becomes:

$$[\frac{\partial P}{\partial T}+\frac{\partial P}{\partial q}\frac{\partial q}{\partial T}]e^{-rT}-re^{-rT}P(T,q(T))=0 \tag{A.2}$$

(A.2) can be reduced to:

$$\frac{\partial P}{\partial T}-rP(T,q(T))+\frac{\partial P}{\partial q}\frac{\partial q}{\partial T}=0 \tag{A.3}$$

Rewriting (A.3) gives:

$$\frac{\frac{\partial P}{\partial T}}{P(T,q(T))}=r-\frac{\frac{\partial P}{\partial q}\frac{\partial q}{\partial T}}{P(T,q(T))} \tag{A.4}$$

1.2. A social optimum

The social planner problem is:

$$Max[P(T,q(T))e^{-rT}-\int_0^T e^{-rx}D(x)dx] \tag{A.5}$$

Using T as control variable, the first-order condition is:

$$[\frac{\partial P}{\partial T}+\frac{\partial P}{\partial q}\frac{\partial q}{\partial T}]e^{-rT}-re^{-rT}P(T,q(T))-e^{-rT}D(T)=0 \tag{A.6}$$

(A.6) can be rewritten as:

$$\frac{\partial P}{\partial T}-rP(T,q(T))+\frac{\partial P}{\partial q}\frac{\partial q}{\partial T}-D(T)=0 \tag{A.7}$$

We can also rewrite (A.7) as:

$$\frac{\frac{\partial P}{\partial T}}{P(T,q(T))}=r-\frac{\frac{\partial P}{\partial q}\frac{\partial q}{\partial T}}{P(T,q(T))}+\frac{D(T)}{P(T,q(T))} \tag{A.8}$$

1.3. Optimal regulation

By comparing (A.3) and (A.7), we find that the optimal marginal tax becomes:

$$\frac{\partial U}{\partial T} = D(T) \tag{A.9}$$

2. Relaxing some of the basic assumptions

2.1. Incorporating pollution

2.1.1. A private optimum

The aquaculture producer now solves:

$$Max[P(T, q(T), s_t)e^{-rT}] \tag{A.10}$$

With T and s_t as control variables, the following first-order conditions are reached:

$$[\frac{\partial P}{\partial T} + \frac{\partial P}{\partial q}\frac{\partial q}{\partial T}]e^{-rT} - re^{-rT}P(T, q(T), s_t) = 0 \tag{A.11}$$

$$\frac{\partial P}{\partial s_t}e^{-rT} = 0 \tag{A.12}$$

(A.11) and (A.12) can be reformulated as:

$$\frac{\partial P}{\partial T} - rP(T, q(T), s_t) + \frac{\partial P}{\partial q}\frac{\partial q}{\partial T} = 0 \tag{A.13}$$

$$\frac{\partial P}{\partial s_t} = 0 \tag{A.14}$$

To find the private optimal pollution level, $\bar{s}$, and rotation time, , T_p*, we need to solve (A.13) and (A.14) as two equations with two unknowns.

2.1.2. A social optimum

Now the objective function is:

$$Max[P(T, q(T), s_t)e^{-rT} - D(s_t)e^{-rT}] \tag{A.15}$$

When using T and s_t as control variables, the first-order conditions are obtained:

$$[\frac{\partial P}{\partial T} + \frac{\partial P}{\partial q}\frac{\partial q}{\partial T}]e^{-rT} - re^{-rT}P(T, q(T), s_t) + re^{-rT}D(s_t) = 0 \tag{A.16}$$

$$(\frac{\partial P}{\partial s_t} - D'(s_t))e^{-rT} = 0 \tag{A.17}$$

(A.16) and (A.17) can be reduced to:

$$\frac{\partial P}{\partial T} - rP(T, q(T), s_t) + \frac{\partial P}{\partial q}\frac{\partial q}{\partial T} + rD(s_t) = 0 \qquad \text{(A.18)}$$

$$\frac{\partial P}{\partial s_t} - D'(s_t) = 0 \qquad \text{(A.19)}$$

To find the social optimal pollution level, s_t*, and rotation time, , T_s*, we have two solve (A.18) and (A.19) as two equations with two unknowns. Furthermore, comparing (A.14) and (A.19), we observe that $s_t* < \bar{s}$, and by comparing (A.13) and (A.18), we find that $T_s* > T_p*$.

2.1.3. Optimal regulation

Since $T_s* > T_p*$, we need a marginal subsidy to increase the rotation time. Comparing (A.14) and (A.18), the marginal subsidy is:

$$-\frac{\partial U}{\partial T} = rD(s_t) \qquad \text{(A.20)}$$

Because $s_t < \bar{s}$ a tax on pollution is required to adjust behavior, and by comparing (A.13) and (A.18), we get the following marginal tax:

$$\frac{\partial V}{\partial s_t} = D'(s_t*) \qquad \text{(A.21)}$$

(A.21) is a Pigovian tax.

2.2. Fixed costs

2.2.1. A private optimum

The aquaculture producer now solves:

$$Max[(P(T, q(T)) - FP)e^{-rT}] \qquad \text{(A.22)}$$

With T as control variable, the first-order condition is:

$$[\frac{\partial P}{\partial T} + \frac{\partial P}{\partial q}\frac{\partial q}{\partial T}]e^{-rT} - re^{-rT}(P(T, q(T)) - FP) = 0 \qquad \text{(A.23)}$$

(A.23) can be written as:

$$\frac{\partial P}{\partial T} - rP(T, q(T)) + rFP + \frac{\partial P}{\partial q}\frac{\partial q}{\partial T} = 0 \qquad \text{(A.24)}$$

2.2.2. A social optimum

The social planner objective function becomes:

$$Max[(P(T, q(T)) - FS)e^{-rT} - \int_0^T e^{-rx}D(x)dx] \qquad \text{(A.25)}$$

With T as control variable, the first-order condition is:

$$[\frac{\partial P}{\partial T}+\frac{\partial P}{\partial q}\frac{\partial q}{\partial T}]e^{-rT}-re^{-rT}(P(T,q(T))-FS)-e^{-rT}D(T)=0 \tag{A.26}$$

(A.26) reduces to:

$$\frac{\partial P}{\partial T}-rP(T,q(T))+\frac{\partial P}{\partial q}\frac{\partial q}{\partial T}+rFS-D(T)=0 \tag{A.27}$$

2.2.3. Optimal regulation

By comparing (A.24) and (A.27), we find the marginal tax:

$$\frac{\partial U}{\partial T}=D(T)-r(FS-FP) \tag{A.28}$$

2.3. Running costs

2.3.1. A private optimum

The aquaculture producer now maximize:

$$Max[R(T,q(T))e^{-rT}-\int_0^T e^{-ry}CP(y)dy] \tag{A.29}$$

Using T as control variable, the following first-order condition applies:

$$[\frac{\partial R}{\partial T}+\frac{\partial R}{\partial q}\frac{\partial q}{\partial T}]e^{-rT}-re^{-rT}(R(T,q(T))-CP(T))=0 \tag{A.30}$$

(A.30) can be written as:

$$\frac{\partial R}{\partial T}-rR(T,q(T))+\frac{\partial R}{\partial q}\frac{\partial q}{\partial T}-CP(T)=0 \tag{A.31}$$

2.3.2. A social optimum

The present value of the current and future welfare becomes:

$$Max[R(T,q(T))e^{-rT}-\int_0^T e^{-ry}CS(y)dy-\int_0^T e^{-rx}D(x)dx] \tag{A.32}$$

With T as control variable, the first-order condition is:

$$[\frac{\partial R}{\partial T}+\frac{\partial R}{\partial q}\frac{\partial q}{\partial T}]e^{-rT}-re^{-rT}R(T,q(T))-e^{-rT}CS(T)-e^{-rT}D(T)=0 \tag{A.33}$$

(A.33) reduces to:

$$\frac{\partial R}{\partial T} - rR(T, q(T)) + \frac{\partial R}{\partial q}\frac{\partial q}{\partial T} - CS(T) - D(T) = 0 \qquad \text{(A.34)}$$

3.3.3. Optimal regulation

By comparing (A.31) and (A.34), we get the marginal tax:

$$\frac{\partial U}{\partial T} = D(T) + CS(T) - CP(T) \qquad \text{(A.35)}$$

2.4. Another formulation of quality

2.4.1. A private optimum

The aquaculture producer now solves:

$$Max[(RP(T) - C(T))e^{-rT}] \qquad \text{(A.36)}$$

The following first-order condition is obtained:

$$[RP'(T) - C'(T)]e^{-rT} - re^{-rT}(RP(T) - C(T)) = 0 \qquad \text{(A.37)}$$

(A.37) can be written as:

$$RP'(T) - C'(T) - rRP(T) + rC(T) = 0 \qquad \text{(A.38)}$$

2.4.2. A social optimum

The discounted welfare maximization problem is:

$$Max[(RS(T) - C(T))e^{-rT} - \int_0^T e^{-rx} D(x)dx] \qquad \text{(A.39)}$$

Using T as control variable, we obtain the following first-order condition:

$$[RS'(T) - C'(T)]e^{-rT} - re^{-rT}(RS(T) - C(T)) - e^{-rT}D(T) = 0 \qquad \text{(A.40)}$$

(A.40) reduces to:

$$RS'(T) - C'(T) - rRS(T) + rC(T) - D(T) = 0 \qquad \text{(A.41)}$$

2.4.3. Optimal regulation

Now from (A.38) and (A.41), we get the marginal tax:

$$\frac{\partial U}{\partial T} = D(T) - RS'(T) + RP'(T) + r(RS(T) - RP(T)) \qquad \text{(A.42)}$$

2.5. Private and social discount rates

2.5.1. A private optimum

The aquaculture producer maximizes:

$$Max[P(T,q(T)e^{-r_pT}]$$ (A.43)

With T as control variable, the first-order condition is:

$$[\frac{\partial P}{\partial T}+\frac{\partial P}{\partial q}\frac{\partial q}{\partial T}]e^{-r_pT}-r_pe^{-r_pT}P(T,q(T)=0$$ (A.44)

(A.44) can be written as:

$$\frac{\partial P}{\partial T}-r_pP(T,q(T))+\frac{\partial P}{\partial q}\frac{\partial q}{\partial T}=0$$ (A.45)

2.5.2. A social optimum

The social planner's objective function becomes:

$$Max[P(T,q(T)e^{-r_sT}-\int_0^T e^{-r_sx}D(x)dx]$$ (A.46)

The first-order condition is:

$$[\frac{\partial P}{\partial T}+\frac{\partial P}{\partial q}\frac{\partial q}{\partial T}]e^{-r_sT}-r_se^{-r_sT}P(T,q(T)-e^{-r_sT}D(T)=0$$ (A.47)

(A.47) reduces to:

$$\frac{\partial P}{\partial T}-r_sP(T,q(T))+\frac{\partial P}{\partial q}\frac{\partial q}{\partial T}-D(T)=0$$ (A.48)

2.5.3. Optimal regulation

By comparing (A.45) and (A.48), we obtain the following marginal tax:

$$\frac{\partial U}{\partial T}=D(T)+(r_s-r_p)P(T,q(T))$$ (A.49)