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Parametric and Semiparametric Estimation of Multi-output Stochastic Ray Production Frontiers

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Measuring the Technical Efficiency of Farms Producing Environmental Output: Parametric and Semiparametric Estimation of Multi-output Stochastic Ray Production Frontiers.

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Abstract

This paper investigates the technical efficiency of Polish dairy farms producing environmental output using the stochastic ray function to model multi-output – multi-input technology. Two general models are considered. One which neglects the provision of environmental output and one which accounts for such output. Three different proxies of environmental output are discussed: the ratio of permanent grassland (including rough grazing) to total agricultural area, the total area of permanent grassland and the amount of environmental subsidies which farmers are paid for providing environmental goods and services. These proxies are discussed on the basis of microeconomic production theory and are empirically compared by the econometric approach using parametric and semiparametric stochastic frontier models.

The main focus is on the estimation of technical efficiency of farms producing the environmental output. Since some farms do not provide such output at all, the stochastic ray frontier functions are estimated to overcome the problem of the zero valued dependent variables which often occur when the Translog output distance function is used.

The detailed results of the technical efficiency analysis show that, although the estimated efficiencies from the models which neglect the environmental output and those which account for the output are rather similar on average, the rankings based on these efficiencies differ. Finally, based on the theoretical economic reasoning and empirical application, we find that, for the given dataset, the semiparametric stochastic frontier model which uses a quantity of permanent grassland area as a proxy of environmental output, is the most suitable for application.

Keywords: environmental output, stochastic frontier analysis, stochastic ray function, Translog, Polish dairy farms

JEL codes: C14, D24, Q12, Q57

1. Introduction

The main purpose of agricultural production is to provide food, fibre, fodder and more recently also energy (i.e. bio-fuels). It is a well known fact that agricultural production has a non-neutral impact on the environment and results in both negative and positive externalities. The negative externalities are those related to the pollution of water, soil and air, but also to the negative impact on biodiversity e.g. due to the monoculture of intensive farming systems and the use of pesticides and herbicides. The positive externalities are related to the goods and services which farmers provide as a by-product of their marketable production which result in, e.g. agricultural landscapes (both in the cultural and ecological sense), richer biodiversity, etc. In economics, both the positive and negative externalities of agricultural production can be classified as public goods. The concept of public goods (or collective goods) was introduced by Samuelson (1954). A good is a public good if it is non-excludable and non-rivalled in consumption. Since there is no market for public goods, this leads to a loss of economic efficiency giving governments an argument to intervene in order to internalise the externalities (Areal, Tiffin and Balcombe, 2012). In the European Union (EU), the governments of member states intervene in the market through agricultural support programs. Environmental measures have been included in the Common Agricultural Policy (CAP) of the EU since the 1990s. Previously, CAP supported prices led to the intensification and industrialisation of farming systems and resulted in both the overproduction of agricultural products and negative effects on the environment.

Because a natural trade-off exists between marketable outputs (animal and crop products) and environmental outputs (e.g. biodiversity, agricultural landscape, etc.), farmers who provide the latter can obtain compensation payments (e.g. environmental subsidies). This is legitimised by the lower productivity of environmentally friendly agricultural production systems. Therefore, it is of high importance for policy makers (but also for tax-payers and farmers) to target the right beneficiaries of environmental subsidies. In productivity or efficiency analysis, most previous studies, which incorporate the environmental outputs, consider the bad output (negative externalities), whereas only a few have focused on the good outputs (positive externalities). A recent literature review of the research on positive externalities in agriculture is provided in Areal, Tiffin and Balcombe (2012).

Since environmental output is defined quite broadly, the perfect measure of environmental output at the farm should reflect different aspects, such as biodiversity, agricultural landscape, and the farm's footprint on water, soil and air, etc. However, such information is often unavailable in economic data at the farm level.

Therefore, a proxy of environmental output is often used. Recently Areal, Tiffin and Balcombe (2012) proposed a very straightforward indicator of environmental output produced at the farm, namely the ratio of permanent grassland to total agricultural area.

However, in this paper, based on the theoretical economic reasoning supported by an empirical application, I show that this specification of the environmental output may result in severe problems both in the econometric estimation and the economic interpretation. Therefore, I suggest the reformulation Areal, Tiffin and Balcombe (2012) approach to environmental output and express this with a proxy of permanent grassland area. This approach does not have the drawbacks of the one proposed by Areal, Tiffin and Balcombe (2012) and provides results very similar to the definition of the environmental output proposed by (Peerlings and Polman, 2004), who suggested measuring the positive environmental output as environmental subsidies paid to the farmer for providing environmental goods and services.

Furthermore, the analysis in this paper differs from Areal, Tiffin and Balcombe (2012) both in applied economic and econometric methodologies. Areal, Tiffin and Balcombe (2012) used Bayesian techniques to estimate the output distance function of the Translog functional form for a sample of dairy farms in England and Wales. I use a different specification of the multi-output technology – the stochastic ray approach introduced by Löthgren (1997, 2000). The use of the stochastic ray approach is motivated by the problem of zero values of output variables observed for some of analysed farms (e.g. dairy farms which do not produce crop output and/or do not provide environmental output). Additionally, I use a different econometric technique, namely nonparametric kernel regression, in order avoid the assumption regarding the specification of the functional form of the underlying technology.

The remainder of the paper is organized as follows. Section 2 discusses the different strategies to measure environmental output. Section 3 presents different approaches to the estimation of the multiple-output – multiple-input technologies. Section 4 focuses on the estimation of stochastic frontier models in parametric and nonparametric regression frameworks and describes the theoretical economic model used in the analyses. Section 5 describes the data and presents the results of the analysis. Finally, section 6 concludes.

2. Measurement of the environmental output

The idea of sustainable agriculture, and therefore also the provision of positive environmental externalities at the farm level, plays an important role in the current CAP. However, only a few studies exist in the production economics literature that analyse the provision of positive environmental output.

Environmental output can be defined either as a positive (desirable) or negative (undesirable) output. In the productivity and efficiency literature, environmental output is most often studied as negative externalities (or undesirable outputs) (e.g. Reinhard, Lovell and Thijssen, 2000; Lansink and Peerlings, 1997; Färe, Grosskopf and Hernandez-Sancho, 2004; Kuosmanen and Kortelainen, 2004, to mention only a few). In analyses of the agricultural sector, environmental (bad) output is often defined as being a negative impact of agricultural production on the soil and water and is usually measured as a surplus of chemical elements such as nitrogen, phosphorus and potassium. An interesting approach to dealing with undesirable outputs has been proposed by Asmild and Hougaard (2006), who considered nutrient removal as a desirable output.

More recently, environmental output has been considered as a positive externality: (e.g. Peerlings and Polman, 2004; Sipiläinen, Marklund and Huhtala, 2008; Areal, Tiffin and Balcombe, 2012; Solovyeva and Nuppenau, 2013). Peerlings and Polman (2004) analysed the wildlife and landscape services produced by Dutch dairy farms, where the positive output was measured as the compensation paid by the government to farms to stimulate wildlife and landscape friendly actions (e.g. postponing the mowing of grass to protect nests of meadow birds). Sipiläinen, Marklund and Huhtala (2008) used a crop diversity index to study the efficiency of organic and conventional dairy farms in Finland, while Solovyeva and Nuppenau (2013) used the biodiversity index to investigate the environmental efficiency of farms in the Ukraine . Areal, Tiffin and Balcombe (2012) used the ratio of permanent grassland to total agricultural area as a proxy for environmental output. Although this indicator is not as precise as, e.g. biodiversity indicators, its advantage is that it can be easily obtained for most datasets at no cost.

In this paper, I consider environmental output as a positive (desirable) output. Since the data on biodiversity is not available in my data set, I decided to use basic proxies of environmental outputs, similar to the one proposed by (Peerlings and Polman, 2004) and (Areal, Tiffin and Balcombe, 2012). Furthermore, I suggest modifying the proxy proposed by (Areal, Tiffin and Balcombe, 2012) and expressing environmental output solely as the area of permanent grassland .

3. Multi-output – multi-input technologies

3.1. Output distance functions

The analysis of agricultural production together with environmental outputs calls for a method which allows for multiple inputs and multiple outputs. In the economic literature, the concept of the distance function is often used to account for multiple output technologies. Input and output distance functions have been introduced by Shephard (1953) and Shephard (1970), respectively. This method has become a standard econometric approach to analyse multi-output – multi-input technologies.

In the econometric estimation of the output distance function, the quantity of one of the outputs is often used as a dependent variable while the remaining output quantities are normalised and then used as explanatory variables along with the input quantities. In the parametric econometric approach, the Translog functional form is frequently used to reflect the shape of the underlying technology.

3.2. Stochastic Ray Function

There is a major problem with the parametric estimation of conventional distance functions which arises when there are zero values in the output variables for some observations and the Translog functional form is used[1]. This paper focuses on the analysis of farm productivity with accounting for environmental output. Since the provision of environmental goods and services is not the main interest of the producer (in contrast to the provision of output which can be sold at market), it is very likely that a substantial share of producers will not produce environmental outputs. In such cases, the problem of zero values in output variables becomes even more significant when the environmental output is considered and the Translog functional form is used to estimate the distance function. However, the use of conventional distance functions requires some *ad hoc* adjustments if there are zero output quantities (e.g. using the method proposed by Battese (1997) or neglecting the observations at which this problem occurs). The alternative approach to the estimation of the multiple-output technologies is the stochastic ray function proposed by Löthgren (1997, 2000). Recently Henningsen, Henningsen and Jensen (2013) demonstrated that the stochastic ray function can model zero output values without any *ad hoc* adjustments or removing observations with zero output quantities. Moreover, the authors used Monte Carlo simulations and showed that the stochastic ray function outperforms the conventional distance function if the share of zero output quantities is large.

In the stochastic ray function specification, the multiple output vector y is decomposed as follows (Löthgren, 2000):

$$y = \ell \cdot m(\theta) \tag{1}$$

The scalar component ℓ is defined as:

$$\ell = \ell(y) = \|y\| \tag{2}$$

where $\|y\|$ is the Euclidean norm ($\|y\| = (\sum_{i=1}^{p} y_i^2)^{1/2}$ of the output vector y. The function $m : [0, \pi/2]^{p-1} \rightarrow [0, 1]^p$, defined by:

$$m_i(\theta) = \cos\theta_i \prod_{j=0}^{i-1} \sin\theta_j \qquad i = 1, ..., p \tag{3}$$

[1] Although, Färe, Martins-Filho and Vardanyan (2010) and Chambers et al. (2013) recently showed the superiority of the generalized quadratic functional form over the Translog functional form, the latter is still the most frequently used in applied productivity and efficiency analysis.

where $\theta \in [0, \pi/2]^{p-1}$ and $\sin\theta_0 = \cos\theta_p = 1$ transforms the polar coordinate angle vector θ to the output mix vector $m(\theta) = y/\ell(y)$ with the norm $\|m(\theta)\| = 1$

$$\theta_i(y) = \cos^{-1}\left(y_i/\|y\| \prod_{j=0}^{i-1} \sin\theta_j\right) \qquad i = 1, ..., p \tag{4}$$

Using the above-described decomposition of the output vector, a multi-input multi-output production technology can be represented as:

$$\ln\|y\| = \ln f(x, \theta), \tag{5}$$

where $f(.)$ is the stochastic ray production function.

By introducing technical inefficiency in terms of the output distance measure, the stochastic ray production function can be related to the output distance function in the following way:

$$\ln D_o = \ln\|y\| - \ln f(x, \theta), \tag{6}$$

where D_o is the output distance measure.

4. Parametric and Semiparametric Stochastic Ray Frontier

Löthgren (1997) proposed estimating the stochastic ray frontier of the Translog functional form specified as:

$$\begin{aligned}\ln(\|y\|) = \alpha_0 + \sum_{m=1}^{M-1} \alpha_m\theta_m + 0.5\sum_{m=1}^{M-1}\sum_{j=1}^{M-1}\alpha_{mj}\theta_{mj} + \sum_{n=1}^{N}\beta_n \ln(x_k) \\ +0.5\sum_{n}^{N}\sum_{k}^{N}\beta_{nk}\ln(x_n)\ln(x_k) + \sum_{m=1}^{M-1}\sum_{n}^{N}\zeta_{nk}\theta_n\ln(x_k) + \epsilon,\end{aligned} \tag{7}$$

where $\epsilon = v - u$ is a composed error term with noise component v and inefficiency component $u = -\ln D_o$ using the standard stochastic frontier framework proposed by Aigner, Lovell and Schmidt (1977) and Meeusen and van den Broeck (1977). The usual distribution assumptions are made (i.e., $v \sim N(0, \sigma_v^2)$, and, $u \sim N^+(0, \sigma_u^2)$) and the frontier model is fitted with maximum likelihood estimation (MLE).

This paper contributes to the existing literature with a semiparametric estimation of the stochastic ray production function using the approach proposed by Fan, Li and Weersink (1996) .

Fan, Li and Weersink (1996) suggested estimating the semiparametric stochastic frontier models in a two step procedure. In the first step, the average production function:[2]

$$y = f(x) + \epsilon \tag{8}$$

is fitted with nonparametric regression methods (e.g. local-linear kernel regression). The local-linear kernel regression is a method of smoothing the conditional expectation of the dependent variable given the set of explanatory variables. This is given by the smooth curve obtained by applying a weighted linear regression at each observation, where the weights of the other observations decrease with the distance from the respective observation. These weights depend on two factors: the kernel function (weighting function) and a set of bandwidth parameters, which need to be specified. The choice of the kernel function is of minor importance (e.g. Silverman, 1986; Taylor, 1989; Racine and Li, 2004; Czekaj and Henningsen, 2013b). The bandwidths were initially determined using a rule of thumb. However, due to recently increased computing power, they can be selected according to data driven bandwidth selection methods (e.g. according to the expected Kullback-Leibler cross-validation criterion (Hurvich, Simonoff and Tsai, 1998)). When the data driven bandwidths selection is applied, the overall shape of the regression function is determined by the data without being restricted by an arbitrarily chosen functional form. The convenient feature of the local-linear kernel estimator is that it automatically provides the estimator of the first derivative of the unknown regression function Li and Racine (2007). Moreover, the nonparametric regression method proposed by Li and Racine (2004) and Racine and Li (2004) that can handle both continuous and categorical explanatory variables can be applied in order to include the categorical regressors (i.e. Z-variables) in the estimation of the average production function.

Henningsen and Henning (2009) showed that the monotonicity property is particularly important for estimating the efficiencies of individual firms in stochastic frontier analysis. In a nonparametric regression setting, the monotonicity condition can be imposed using the constraint weighted bootstrapping (CWB) method proposed by Hall and Huang (2001) and extended by Du, Parmeter and Racine (2013). For a recent application of this approach in stochastic frontier analysis see: Parmeter and Racine (2013) and Parmeter et al. (2013).

In the second step of the Fan, Li and Weersink (1996) procedure, residuals of the nonparametrically estimated average production frontier ($\hat{\epsilon}$), are decomposed into a constant,

[2] Fan, Li and Weersink (1996) define the frontier model $g(x) = f(x) + \mu$, where $g(x)$ is a frontier production function, $f(x) = E[y|x]$ is a average production function and error term $\xi = y - g(x) = \epsilon - \mu$ with $E[\xi|x] \neq 0$. I use a modification of the Fan, Li and Weersink (1996) approach suggested by Henningsen and Kumbhakar (2009), where the average production function is defined as $f(x) = E[y|x]$ and a corresponding error term $\epsilon = y - f(x) = y - E[y|x]$ with $E[\epsilon|x] = 0$.

(μ), statistical noise, (v), and inefficiency, (u), terms in the following way:

$$\hat{\epsilon} = \mu + v - u. \tag{9}$$

Henningsen and Kumbhakar (2009) proposed estimating the Fan, Li and Weersink (1996) model in logarithmic output and input quantities.

$$\ln y = \ln f(x) + \epsilon \tag{10}$$

This modification of the Fan, Li and Weersink (1996) approach is useful in the nonparametric regression framework due to several reasons[3]. First, it allows the use of the usual specification of a stochastic frontier function, where the dependent variable is logarithmic so that the predicted dependent variables can not be negative. Second, the log-transformed values of output and input quantities are more equally Distributed, which is particularly desirable when fixed bandwidths in local-linear kernel regression are used. Third, the unknown true relationship between the input quantities and the output quantity is likely much closer to a log-linear relationship (Cobb-Douglas technology) than a linear relationship (linear technology) so that the use of logarithmic quantities of the inputs and the output allows for larger bandwidths, which in turn increases the precision of the local-linear estimates, because they are based on a larger number of observations. Fourth, estimated gradients of input variables in the nonparametric regression can be directly interpreted as distance elasticities with respect to input quantities.

4.1. Z-variables in Semiparametric Stochastic Ray Frontier

The estimation of technical efficiency has been of primary interest in the productivity and efficiency analysis literature since the seminal work of Farrell (1957). More recently, attention has been paid to the role of exogenous factors (often referred to as environmental variables or Z-variables[4]) that may influence either productivity or (in)efficiency (Sun and Kumbhakar, 2013). Because there is no general principle that the Z-variables influence the productivity or (in)efficiency (or both), therefore the number of different approaches has been proposed in the literature. Kumbhakar (1990) and Battese and Coelli (1992) proposed a multiplicative decomposition of the inefficiency term as a function of time (in panel data context) or as a function of Z-variables in a cross-section setting (Sun and Kumbhakar, 2013). This specification is referred to as a scaling property of inefficiency (e.g. Alvarez et al., 2006; Simar, Lovell and Eeckaut, 1994; Wang and Schmidt, 2002). Alternatively, Kumbhakar, Ghosh and McGuckin (1991), Huang and Liu (1994) and Battese and Coelli (1995) proposed the additive decomposition of technical inefficiency.

[3] See e.g. Henningsen and Kumbhakar (2009) and Czekaj and Henningsen (2013b) for details.
[4] Since this paper aims to analyse efficiency in the presence of environmental output, the notion of Z-variables instead of environmental variables is used to avoid confusion.

The alternative approach to handling the presence of Z-variables is to include them as shift variables which directly influence the core of the frontier function.

Recently, Zhang (2012), Sun and Kumbhakar (2013) and Parmeter, Wang and Kumbhakar (2013) showed how to account for the presence of Z-variables in semiparametric stochastic frontiers. Zhang (2012) estimated the semiparametric smooth coefficient stochastic frontier model using Z-variables as shift variables which directly influence the production frontier. Sun and Kumbhakar (2013) extended the model proposed by Zhang (2012) incorporating the Z-variables not only in the core of the production frontier, but also in the technical inefficiency part of the stochastic frontier model. Parmeter, Wang and Kumbhakar (2013) used the scaling property of inefficiency. In their approach, inefficiency is explained as a nonparametric function of Z-variables, whereas the production function has a parametric structure. However, it needs to be noted that, although Zhang (2012), Sun and Kumbhakar (2013) and Parmeter, Wang and Kumbhakar (2013) used nonparametric methods, they still imposed some parametric structure on the underlying technology in their models.

In this paper, I use the fully nonparametric estimate of the (average) production technology in order to obtain the semiparametric stochastic frontier model, which does not depend on the specific functional form of the underlying technology. In the analyses included in this paper, I make the assumption that the Z-variables influence productivity and not the inefficiency[5]. Therefore, the Z-variables are included in the estimation of the nonparametric stochastic ray production function, which is then used to obtain the semiparametric stochastic ray frontier model. In the parametric stochastic frontier, the Z-variables are also included in the regression function together with the remaining explanatory variables.

4.2. The Economic Model

Two general economic models are used in this paper. The first model (referred to as Model I, hereafter) only accounts for conventional market outputs, whereas the second model (Model II, thereafter) also incorporates the provision of environmental goods and services as additional (environmental) output. Three different specifications of environmental output are considered, therefore Model II is specified in three variants: A, B and C. In the Model II A, environmental output is specified following Areal, Tiffin and Balcombe (2012) as the ratio of permanent grassland to total agricultural area. In the Model II B, the area of permanent grassland is used as a proxy for environmental output. Finally, in the Model II C, environmental output is defined following Peerlings and Polman (2004)

[5] The Z-variables considered in this paper influence the core of the frontier (e.g. regional differences in soil quality, climate, etc.) rather than the inefficiency. Moreover, according to my best knowledge, a semiparametric inefficiency effect model has not yet been developed, but the formulation of such a model is beyond the scope of this paper

as the compensation paid to the farmer (as the environmental subsidies) for providing environmental goods and services.

The economic model is described using the concept of the Shephard (1970) distance function[6].

If the production possibility set, $P(x)$, is defined as:

$$P(x) = \{y : x \text{ can produce y } \} = \{y : (x, y) \in S\} \tag{11}$$

where, $y = (y_1, ...y_m) \in R^m_+$ is a non-negative vector of m outputs, $x = (x_1, ...x_n) \in R^n_+$ is a non-negative vector of n inputs and S denotes the technology set. Furthermore, it is assumed that $P(x)$ is a compact set (i.e. it is convex, closed and bounded), $0 \in P(x)$ (inaction is possible) and $P(0) = 0$ (it is impossible to produce positive output without using a positive level of inputs) and both inputs and outputs are freely disposable. Then the output distance function, $D_O(x, y)$, is defined on the output set, $P(x)$, as:

$$D_O(x, y) = \inf\{\delta : \frac{y}{\delta}) \in P(x)\}. \tag{12}$$

Figures 1 and 2 illustrate the multi-output technologies considered in Model I and Models II A, B and C, respectively.

Model I, presented in figure 1, is an illustration of the conventional distance function representation of multiple output technology. For simplicity, only two market outputs are considered: animal output (YA) and crop output (YC). It can be easily seen that the production possibility set ($P(x)$) satisfies the standard axioms presented above. The boundary of $P(x)$, ranging from point A to point B, is the production possibility curve (transformation curve) which represents combinations of maximum attainable output quantities.

Under the assumption of constant returns to scale, scaling all input quantities by a constant will result in a proportional shift in the production possibility curve. This is illustrated by the production possibility set ($P(2x)$) which represents the production possibility set ($P(x)$) scaled by the factor $k = 2$.

An illustration of the distance function representation of multiple output technology in the presence of the environmental output (desirable output) is presented in figure 2. For illustration purposes, market outputs: animal output (YA) and crop output (YC) are aggregated to one output (denoted by Y) on the vertical axis. Environmental output is denoted by YE and is depicted on the horizontal axis.

When environmental output is considered in Model II, the production possibility frontier might not be a smooth function even though the production possibility set is still

[6] Since the stochastic ray function is a specific mathematical representation of the conventional Shephard (1970) distance function, for the convenience of the reader, in the discussion of the economic model, the the Shephard (1970) formulation is used.

Figure 1: Model I

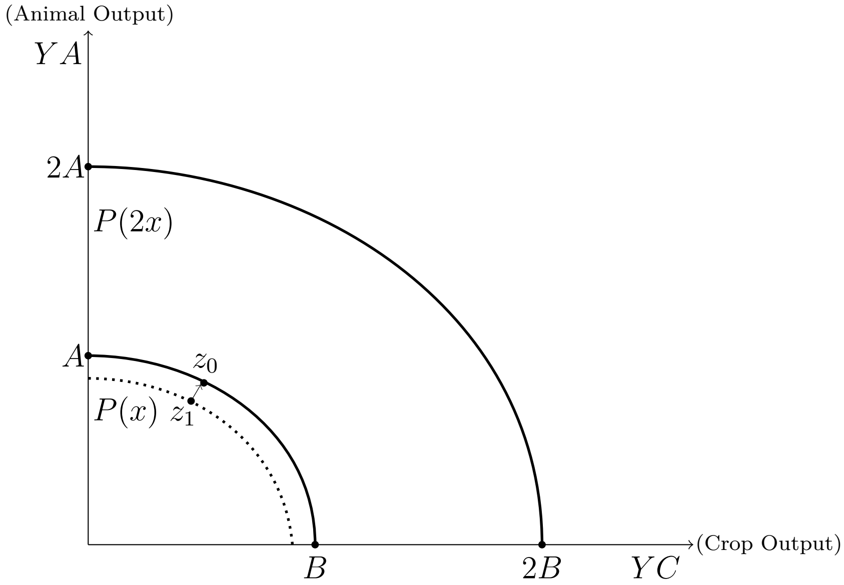
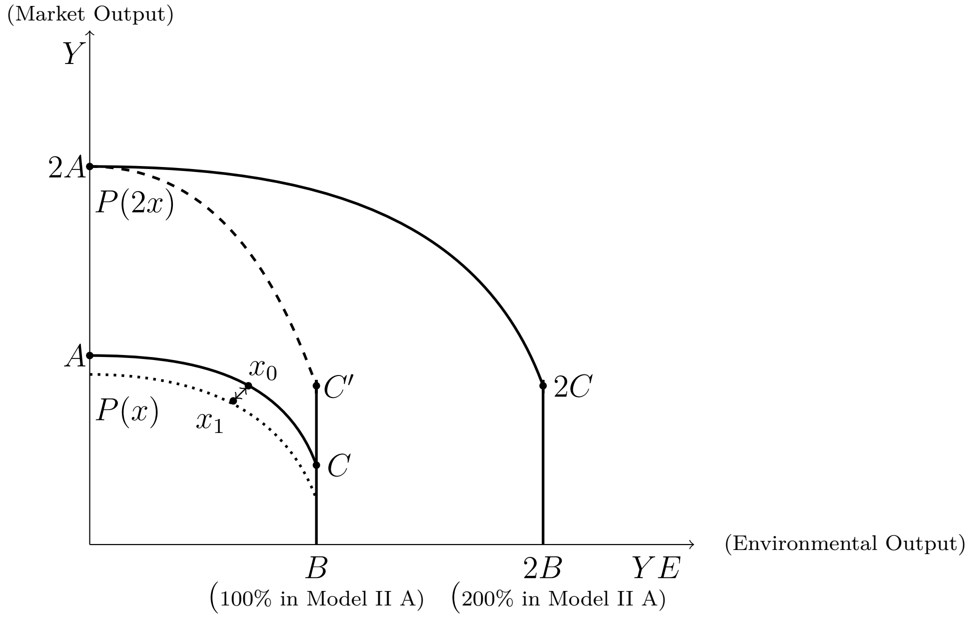


Figure 2: Model II



a compact set (i.e. it is convex, closed and bounded). This could happen if the farmer can still produce a positive amount of market output when the environmental output is maximised (e.g. 100% in the Model II A, the total agricultural area in Model II B, and the maximum attainable amount of environmental payments in Model II C), which is illustrated in figure 2. When the farm does not produce any environmental output, the maximum attainable level of the market output is given by point A. If environmental output is defined as the the ratio of permanent grassland to total agricultural area as a proxy for environmental output the increase in the environmental output will result in lower market output (e.g. due to lower crop output as well as possibly a lower milk yield due to a lower nutrient content of fodder from pastures than e.g. from maize silage). The substitutability between market output and environmental output is illustrated by the smooth curve of the production possibility frontier ranging from point A to point B. However, even when environmental output is maximized, it is still possible to produce a positive quantity of market output. Therefore, beyond point B, the production possibility frontier has a straight line form (perpendicular to the horizontal axis) extending to point C. The potential "kink" illustrated at point C and the straight line from point C to point B may involve problems in the econometric estimation of the Translog function (or any other smooth and differentiable function) unless the share of farms which provide the maximum level of environmental output is very small, because the true frontier is not smooth and differentiable in such a case.[7]

There is a more important problem which relates to the scaling of production technology, when environmental output is defined as the the ratio of permanent grassland to total agricultural area as a proxy for environmental output(i.e. Model IIA). For simplicity, it is illustrated for constant returns to scale. A proportional increase in all inputs will not cause a proportional increase in environmental output. The production possibility set $P(2x)$ presented in figure 2, which is the production possibility set for a scaled (doubled) input vector, has a different shape for Model II A and for Models II B and C. When environmental output is defined as in Models II B and C, the boundary of the production possibility set $P(2x)$ is given by a smooth line which extends from point 2A to point 2C, and then by a straight line (perpendicular to the horizontal axis) to point 2B. The segment 2C-2B represents all feasible quantities of the market output given the maximised environmental output. However, in the case of Model II A, the production possibility set $P(2x)$ is different, because the environmental output can not be scaled beyond point B. Therefore, the boundary of the production possibility set $P(2x)$ will not range from point 2A to point 2B (through point 2C), but will go to point B through point 2C' as represented by a dashed line in figure 2. In other words, assuming constant returns

[7] This problem is probably less severe in Model II C, because it is less likely that many farmers receive the maximum attainable amount of environmental payments than that many farmers use 100% of their land as permanent grassland.

to scale, when two farms which are characterised by equal input quantities and which produce the same quantity of outputs are merged, the quantity of all outputs, and hence the aggregated output of the merged farm, should double. However, this is not the case in Model II A because the environmental output of these farms (given by the ratio of permanent grassland to total agricultural area as a proxy for environmental output) will remain unchanged. This means that the measures of interest (e.g. distance elasticities and returns to scale) obtained from Model II A may be misleading.

Since the definition of environmental output proposed by Areal, Tiffin and Balcombe (2012) may involve serious problems both in the estimation and interpretation of the econometric model based on the theoretical discussion above, I suggest accounting for environmental output either by using the proxy of the total area of permanent grassland (as in Model II B), or by following Peerlings and Polman (2004), and using the value of environmental subsidies (Model II C).

The economic interpretation of Model II B and II C is similar to Model I, which does not include environmental output. In contrast to Model II A, outputs in these models can be radially scaled under constant returns to scale. Therefore, if the $P(x)$ (and its scaled counterpart $P(2x)$) satisfies the standard axioms, the production possibility frontier (representing the substitutability between the market output (Y) and the environmental output (YE)) is given by the smooth curve ranging from point A through point C to point B (and from point 2A through point 2C to point 2B if the input quantities are scaled by the factor $k = 2$) in figure 2.

Moreover, figures 1 and 2 can be also used to illustrate the role of the Z-variables in Model I and in Models II A–C. In this paper, the Z-variables are included in the stochastic frontier models as shift variables. Therefore, they reflect the possible differences in frontiers (technologies), e.g. between different regions, or between groups of farms (e.g. located in less favoured areas (LFA) and not located in these areas, etc). This is illustrated in figures 1 and 2 where the production frontier (passing through point z_0) has shifted downwards to the origin representing the new production frontier (passing through point z_1, marked by the dotted line) of less productive technology (i.e. technology of the farms located in less productive geographic regions, less favoured areas, etc.).

5. Application

5.1. Data

In this study, I use a cross sectional data set from the Polish Farm Accountancy Data Network (FADN) which consists of 2422 farms specialising in dairy productionin 2010 to investigate the farms' technical efficiency in the presence of environmental output. Farm technology is modelled with the stochastic ray function using nonparametric kernel regression. Two general models are considered: one with two market outputs (animal and crop output[8]) and one with three outputs (two market outputs and one environmental output).

The main market output, i.e. animal output (denoted as YA) is defined as revenue from selling milk, beef and remaining animal production. The remaining output, crop output (denoted as YC), is defined as the revenue from crop production and other outputs (services). Environmental output (denoted as YE) is defined in three different ways: (i) as the ratio of permanent grassland to total agricultural area as a proxy for environmental output in Model II A (denoted as YE_A), (ii) as the number of hectares of permanent grassland in Model II B (denoted as YE_B), (iii) as the total environment subsidies in Model II C (denoted as YE_C).

Four inputs are used in the regression analyses: labour (L), land (A), intermediate inputs (V), and capital (K). Labour is measured by Annual Work Units (AWU), where 1 AWU equals 2200 hours of work. The total utilised agricultural area in hectares is used as a measure of land input. Intermediate inputs are measured as the sum of total farming overheads (e.g. maintenance, energy, services, other direct inputs) and specific costs (e.g. fodder, medicine, fertilisers, etc.). The capital input is measured as the value of total fixed assets excluding the value of land.

Additionally, a set of dummy variables indicating regional location and a dummy variable indicating the location of farms in less favoured areas are included as Z-variables in the econometric models.

Descriptive statistics of the regression variables are presented in Table 1.

5.2. Results

All estimations and calculations were conducted within the statistical software environment "R" (R Development Core Team, 2012) using the add-on package "np" (Hayfield and Racine, 2008) for nonparametric regression and specification tests and the add-on package "frontier" (Coelli and Henningsen, 2013) for stochastic frontier analysis[9].

Kernel regression methods are useful in applied productivity and efficiency analysis primarily because they do not require the assumption regarding the functional form of the relationship between the explanatory variables and the dependent variable. However, there is no reason to apply these methods if the parametric model is correctly specified. Therefore, before I proceed with the nonparametric regression analysis, I tested the validity of the parametric stochastic ray models of the Translog functional form for all four models (Model I and three variants of Model II) with parametric and nonparametric statistical tests. The first test is the standard parametric regression error specification test (RESET) proposed by Ramsey (1969) which tests whether non-linear combinations of the

[8] In the application, services are included in crop output, therefore the more precise term is *other* output. However, for simplicity, it is called *crop* output.

[9] The R commands used for this analysis are available from the author upon request.

Table 1: Descriptive statistics

Variable	Unit	Min	Median	Mean	Max	Std. Dev
YA	k PLN	8.007	113.100	162.100	1842.000	165.969
YC	k PLN	0.136	24.630	36.610	782.900	45.915
YE_A	%	0.000	31.940	33.610	100.000	21.732
YE_B	ha	0.000	7.730	10.680	236.400	12.445
YE_C	k PLN	0.000	0.000	1.841	129.800	5.836
L	AWU	0.430	1.855	1.874	10.180	0.589
A	k PLN	2.080	25.840	32.640	535.800	28.759
V	k PLN	4.720	76.690	106.300	1276.000	103.436
K	k PLN	23.120	384.500	524.800	4562.000	485.413

Dummy Variable	
REG1 1 if the farm is located in Pomorze and Mazury	16%
REG2 1 if the farm is located in Wielkopolska and Śląsk	23%
REG3 1 if the farm is located in Mazowsze and Podlasie	52%
REG4 1 if the farm is located in Małopolska and Pogórze	9%
LFA 1 if the farm is located in a LFA	75%

fitted values enhance the fit of the model (then the model is misspecified) or not (then the model is correctly specified). In the RESET test, F-statistics are used to compare the estimated models. In this paper, I used the conventional RESET test to test the functional form of the Translog average stochastic ray function. The second test is a variation of the RESET test, where the Likelihood Ratio test is used (instead the F-test) to test the Translog (frontier) ray function. Therefore, this test is denoted as RESET-LR and uses χ_2 statistics to test the correct parametric specification of estimated models. Additionally, the average stochastic ray functions of the Translog functional forms were tested using the nonparametric consistent model specification test proposed by Hsiao, Li and Racine (2007). Detailed results of diagnostic tests of the Translog (average and frontier) stochastic ray models are presented in table 2.

According to the results of the conducted RESET tests, the Translog functional form was clearly rejected for all analysed regression models (both the average and frontier) stochastic ray functions. Based on the outcome of the nonparametric consistent model specification test, the Translog functional form is not the correct specification of the regression function for all four estimated models.

The detailed results of the estimation of the parametric stochastic ray frontier models are presented in tables A1 and A2 in the Appendix.

Since the monotonicity condition was violated for some observations in the nonparametric estimations of the stochastic ray models, the constrained weighted bootstrapping

Table 2: Results of diagnostic tests of Translog stochastic ray models

Test	Function		Statistics	Decision
RESET test	Model I		$RESET = 7.892$, $p < 0.001$	rejected
	Model II	A	$RESET = 34.866$, $p < 0.001$	rejected
		B	$RESET = 12.952$, $p < 0.001$	rejected
		C	$RESET = 5.922$, $p = 0.003$	rejected
RESET-LR test	Model I		$\chi_2 = 31.352$, $p < 0.001$	rejected
	Model II	A	$\chi_2 = 140.370$, $p < 0.001$	rejected
		B	$\chi_2 = 53.217$, $p < 0.001$	rejected
		C	$\chi_2 = 30.540$, $p < 0.001$	rejected
Nonparametric consistent model specification test	Model I		$Jn = 1.630$, $p = 0.033$	rejected
	Model II	A	$Jn = 9.859$, $p < 0.001$	rejected
		B	$Jn = 2.259$, $p = 0.005$	rejected
		C	$Jn = 2.337$, $p = 0.008$	rejected

(CWB) method proposed by Hall and Huang (2001) and extended by Du, Parmeter and Racine (2013) was used to impose this condition at all data points[10]. This is a novel and powerful method which allows one to impose multiple shape constraints on the regression models.

The results of the estimation of the nonparametric average stochastic ray models are presented in the Appendix tables A3 – A6, whereas corresponding semiparametric frontier stochastic ray models are presented in the Appendix tables A7 and A8.

According to the bootstrap significance test proposed by Racine (1997) and Racine, Hart and Li (2006), in all estimated models, all explanatory variables (logarithms of input quantities, polar coordinate angle vectors as well as Z-variables) are statistically significant. The large bandwidths of the continuous regressors indicate that these variables (i.e. intermediate inputs in all models except Model II A, and labour in Models II A and II B) enter the nonparametric model linearly. However, this linearity does not imply separability Parmeter et al. (2012). The actual values of the explanatory variables with large bandwidths do not affect the slopes of any regressor, but the slope of the explanatory variables with large bandwidths may depend on the actual values of the explanatory variables with smaller bandwidths. The small bandwidth parameters for remaining continuous variables indicate that estimated models are nonlinear in these variables. All nonparametric

[10] The detailed results of the constrained and unconstrained nonparametric models are depicted in tables A3 – A6 in the Appendix. Both unconstrained and constrained models utilised the same bandwidth parameters. The CWB method is used to ensure that the monotonicity condition is fulfilled at all data points (e.g gradients of logarithmic input quantities of unconstrained models are restricted to be non-negative).

I am grateful to Jeff S. Racine and Christopher F. Parmeter for sharing the R code implementation of the CWB and their valuable suggestions and comments). Although this restriction slightly affects the estimates (gradients) of the remaining regressors (both continuous and categorical), the differences are rather small.

stochastic ray models are highly nonlinear in polar coordinate angle vectors (θ in Model I and θ_1 and $\theta_2$in Models II A–C). The Z-variables in the nonparametric stochastic ray models are smoothed as categorical regressors using appropriate kernel functions. The Z-variables indicating the FADN region and the location of the farm in the LFA are specified as categorical regressors and the Li and Racine (2004) generalisation of the Aitchison and Aitken (1976) kernel is used. The bandwidth parameters for categorical regressors can be between 0 (then the sample is divided into sub-samples based on this category) and 1 (the effect of this variable on the conditional mean of the dependent variable is smoothed out indicating that this variable is irrelevant). The estimated bandwidths for all Z-variables in all estimated models are clearly larger than 0 and smaller than 1. This means that all Z-variables are relevant.

In all estimated parametric and semiparametric stochastic ray approximations of the true frontiers of all considered models, the inefficiency is significant. The average efficiencies from the parametric and the semiparametric stochastic frontier models are comparable. However, the semiparametric models consistently report mean efficiency which is 3 to 4 percentage points higher than the parametric models. Additionally, the mean efficiencies in the models that account for environmental output are higher than in the models that do not account for this output.

Moreover, deviations from the frontier are both due to inefficiency as well as statistical noise, which is indicated by the estimates of the γ parameter. In the semiparametric stochastic frontier models, the γ parameters range between 0.53 and 0.63, whereas in the semiparametric models they range between 0.57 and 0.69. This indicates that in the analysed sample, both inefficiency and stochastic noise play an important role in the explanation of the deviations from the estimated production frontiers.

Table 3 presents the distance elasticities[11] obtained based on the parametric Translog stochastic ray and the semiparametric stochastic ray models, respectively. The calculated distance elasticities with respect to the output quantities can be used to calculate the relative marginal rate of technical transformation (RMRTT)[12], which can be used to determine how much production of one output (e.g. market output) needs to be reduced in order to increase production of the other output (e.g. environmental output).

Following Färe and Primont (1995), the elasticity of scale for the output distance function can be calculated based on the negative sum of the distance elasticities with respect to the input quantities. Furthermore, the relative effects of the inputs on the aggregated

[11] Derivation of the distance elasticities with respect to output quantities for the stochastic ray function with two and three outputs are shown in section B in the Appendix.

[12] The relative marginal rate of technical transformation (RMRTT) can be defined as follows:

$$RMRTT_{y_m,y_n} \equiv \frac{\partial \ln y_m}{\partial \ln y_n} = -\frac{\frac{\partial \ln D_o}{\partial \ln y_n}}{\frac{\partial \ln D_o}{\partial \ln y_m}}, \tag{13}$$

where $\frac{\partial \ln D_o}{\partial \ln y_m}$ denotes the distance elasticity of the mth output.

output can be obtained based on the shares of distance elasticities of the inputs calculated as: $s_n = {}^{\varepsilon_n}/\sum_i^N \varepsilon_i$, where ε_n is the distance elasticity of the nth input.

Table 3: Mean distance elasticities of parametric and semiparametric stochastic ray frontiers

	Parametric				Semiparametric			
	Model I	Model II			Model I	Model II		
Regressor		A	B	C		A	B	C
$\ln(YA)$	0.743	0.646	0.469	0.721	0.729	0.665	0.497	0.690
$\ln(YC)$	0.257	0.222	0.419	0.269	0.271	0.220	0.393	0.298
$\ln(YE)$	-	0.132	0.112	0.010	-	0.115	0.110	0.012
$\ln(L)$	-0.076	-0.068	-0.047	-0.079	-0.113	-0.091	-0.096	-0.114
$\ln(A)$	-0.101	-0.165	-0.333	-0.127	-0.114	-0.170	-0.338	-0.128
$\ln(V)$	-0.756	-0.521	-0.551	-0.728	-0.769	-0.536	-0.546	-0.739
$\ln(K)$	-0.172	-0.134	-0.131	-0.173	-0.163	-0.131	-0.135	-0.171
ε	-1.105	-0.888	-1.062	-1.106	-1.160	-0.923	-1.114	-1.152
$s_{\ln(L)}$	0.069	0.077	0.044	0.071	0.097	0.098	0.086	0.099
$s_{\ln(A)}$	0.091	0.186	0.314	0.115	0.098	0.183	0.303	0.111
$s_{\ln(V)}$	0.684	0.587	0.519	0.658	0.664	0.578	0.490	0.641
$s_{\ln(K)}$	0.156	0.151	0.123	0.156	0.141	0.141	0.121	0.148

In general, the parametric and the semiparametric approach provide similar results. However, there are some differences in the economic interpretation which need to be addressed.

Both semiparametric and parametric models report on average increasing returns to scale given by the mean elasticity of scale equal to 1.105 and 1.160, respectively.

When only market outputs are considered (Model I), the largest relative marginal contribution to the aggregated output is given by the intermediate inputs, which equate to around 66% and 68% of aggregated output in the semiparametric and parametric models. The capital input and the land input account for around 14% and 10% of the aggregated output in the semiparametric model and around 16% and 9% in the parametric model. The relative importance of the labour input is equal to 10% and 7% in the semiparametric and parametric models, respectively.

Distance elasticities with respect to the output quantities estimated for Model II A, in which the ratio of permanent grassland to total agricultural area as a proxy for environmental output is used as an indicator of environmental output following Areal, Tiffin and Balcombe (2012), differ from Model I, no matter which regression method is used. The estimated distance elasticities with respect to market outputs (animal and crop outputs) are lower than in Model I, and are equal to 68% (65%) and 21% (22%) for the semiparametric (parametric) model. The relative marginal contribution of environmental output in the aggregated output is equal to around 11% and 13%, respectively, in the semiparametric and parametric models. The distance elasticities with respect to inputs also differ. The marginal contribution to the aggregated output of the intermediate inputs (land) decreased (increased) once environmental output was accounted for. The relative marginal contribution of the land input is greater in the model that account for environmental output (e.g. Model II A) than in the model that does not account for environmental output (Model I), because an increase in land input (holding all other input quantities constant) results in less intensive production, which allows for greater environmental output, particularly when environmental output is measured as the share (or quantity) of permanent grassland, which is usually less intensively used than arable land. The calculated relative marginal rates of technical transformation between environmental output and animal and crop outputs are -0.2 and -0.59. This means that if a farmer wishes to increase the provision of environmental output by 1%, she needs to sacrifice 0.2% of the value of animal production or 0.6% of the value of crop production. The calculated elasticity of scale for Model II A is lower than in Model I and is equal to 0.92 and 0.89 for semiparametric and parametric models, respectively. Although Areal, Tiffin and Balcombe (2012) do not discuss estimated elasticities of scale for their models, they provide the estimates of the slope coefficients. Based on these coefficients, the elasticity of scale from the model which does not account for environmental output is around 1.19, whereas the elasticity of scale from the model which accounts for environmental output is around 0.77. This means that when only market output is considered (which is most often the case), the analysed farms are too small, which is often confirmed in the literature (e.g. Czekaj and Henningsen, 2013a; Latruffe et al., 2012). However, when environmental output is included in the economic model, the same farms are too large. This finding could be used by policy makers to justify the support of small scale farms. Nevertheless, although this straightforward explanation seems to be reasonable, it is not necessarily true. The specification of environmental output as the ratio of permanent grassland to total agricultural area as a proxy for environmental output has already been discussed in section 2. Based on economic reasoning, I showed that this specification of environmental output leads to severe problems in economic interpretation, particularly when scale economies are considered.

Therefore, two alternative models, with different specifications of the environmental output, were estimated. In the first model (Model II B), environmental output is measured by the area of permanent grassland, whereas in the second model (Model II C), environmental output is expressed by environmental subsidies.

In Model II B, the estimated distance elasticities differ even more noticeably than the distance elasticities in Model I, and those estimated in Model II A. For instance, the relative importance of animal output decreased, while the relative importance of crop output increased compared to Model I. However, the estimates of the distance elasticity with respect to environmental output for Model II B, which is 11% (both for the parametric and semiparametric models), are very similar to the one calculated in Model II A.

The calculated marginal rates of technical transformation between the environmental output and animal and crop outputs are -0.24 and -0.27, respectively. This means that if a farmer wishes to increase the provision of environmental output by 1%, he will have to reduce the value of animal production by 0.24%, or the value of crop production by 0.27%.

Distance elasticities with respect to input quantities for Model II B are different to Model I and Model II A. The main differences are in the estimated elasticities of land input and intermediate inputs. Although the distance elasticity of intermediate inputs is lower than in Model I (and similar to Model II A), the distance elasticity of land input is three times as large as in Model I and twice as large as in Model II A. The higher relative marginal contribution of land input results from the specification of the environmental output, since the quantity of this output is to some extent related to the quantity of land input.

Finally, the estimation results (based on both the parametric and the semiparametric regressions) of the Model II C, in which the value of environmental subsidies is used as a proxy of environmental output, are very similar (according to the obtained distance elasticities as well as the estimated efficiencies and their rankings) to Model I, which does not account for environmental output.

The distance elasticity with respect to the environmental output calculated in Model II C is equal to 1% and it is, therefore, not economically significant. According to this result, farmers can increase environmental output without restricting market output. This mean that farmers obtain environmental subsidies for very low quality land, which they would not use for agricultural production anyway. The important drawback of this proxy is that, if it accounts for the provision of environmental output, it only does it correctly for the farms which have applied for environmental subsidies, as it ignores farms which provide environmental output, but which have not applied for such subsidies. Given that in the analysed sample 21% of farmers obtained environmental subsidies and there was a variation in this variable (coefficient of variation equal to 3.17 for the whole sample and 1.15 for the sub-sample of the farms which obtained environmental subsidies), this proxy is not a good approximation of environmental output.

The estimation results of Model II C are quite similar to Model I. Therefore, the derived economic measures (see Table 3) and the estimated technical efficiencies (see Table 4 and figures A1 – A6 in the Appendix) are very similar in these two models, although it should be noted that they are not equal.

The effect of the dummy variable LFA (indicating whether or not a farm is located in a less favoured area) was negative in all models, no matter which regression method was used. This means that, on average, farms located in less favoured areas were less productive than farms not located in these areas. It is worth adding that in the models estimated using nonparametric regression techniques, although the effect of this variable was significant, the effect was noticeably smaller than in the models estimated with parametric regression techniques.

Moreover, the statistically significant heterogeneity of production technologies was found based on the estimates of the set of dummy variables (in nonparametric regression models the factor was used instead of dummy variables) indicating the regional location of the farms. According to the nonparametric estimates of Model I, farms located in the regions Pomorze and Mazury (REG1), Wielkopolska and Śląsk (REG2), and Małopolska and Pogórze (REG4) are less productive than farms in the region Mazowsze and Podlasie (REG3). When environmental output is included, only farms located in the region Wielkopolska and Śląsk (REG2) are less productive than farms in the region Mazowsze and Podlasie (REG3).

In applied productivity and efficiency studies, authors usually focus on estimated individual efficiency scores. According to the results of conducted stochastic frontier analyses, I found that the average technical efficiency of analysed farms is around 85 - 88%. The estimates of technical efficiency do not differ substantially between The parametric and semiparametric frontier models, or between the models which account for environmental output and those which do not. However, the ranking of farms differs between the model that does not include environmental output and the models which account for this output, no matter how it is specified. This is illustrated by rank correlation coefficients presented in table 4 and scatter plots in figures A2, A4, and A6.

Table 4: Correlation coefficients (Spearman's rank correlation ρ) of estimated efficiencies

	TL MI	TL MII A	TL MII B	TL MII C	SP MI	SP MII A	SP MII B	SP MII C
TL MI	1.000							
TL MII A	0.818	1.000						
TL MII B	0.867	0.934	1.000					
TL MII C	0.979	0.801	0.851	1.000				
SP MI	0.968	0.796	0.843	0.952	1.000			
SP MII A	0.844	0.916	0.922	0.829	0.855	1.000		
SP MII B	0.861	0.902	0.946	0.847	0.873	0.960	1.000	
SP MII C	0.952	0.787	0.836	0.965	0.976	0.843	0.861	1.000

The values of the Spearman's rank correlation coefficients of efficiency scores (i.e. correlations of rankings) within the same economic models estimated with different econometric approaches are equal to around 0.92-0.97, which indicates almost perfect correlation. This means that both estimation approaches (semiparametric and parametric) deliver very similar results.

When the Spearman's rank correlation coefficient is used to compare the efficiency scores obtained from the models that do not include environmental output with those that account for this output, although the correlation is still strong (ρ ranging from 0.82

to 0.98) it is not perfect. This means that for some individual farms, the estimated efficiencies differ.

These findings are similar to the results of Areal, Tiffin and Balcombe (2012), who also did not find considerable differences in estimated average efficiency scores, although they found differences in rankings.

6. Conclusion

I proposed using the nonparametric kernel regression method to estimate semiparametric stochastic ray frontier models to investigate the technical efficiency of farms with environmental output within multi-output – multi-input technology.

The stochastic ray approach was applied rather than the conventional distance function approach to overcome the problem of zero valued output quantities, which arose because some of the analysed farms did not provide environmental output. The conventional parametric specification of stochastic ray models of the Translog functional form were rejected for my dataset both by the parametric and nonparametric statistical test. Therefore, semiparametric stochastic ray frontier models were applied to overcome the problem related to the specification of the shape of the regression function.

Three different proxies of environmental output were considered. Based on the theoretical economic model supported by the empirical econometric application, this paper finds that the recently proposed proxy of environmental output defined as the ratio of permanent grassland to total agricultural area as a proxy for environmental output is not necessarily consistent with microeconomic production theory. The other proxy of environmental output used previously in the literature – the value of environmental subsidies, was not a useful measure for the analysed sample of farms.

The proposed alternative approach to the measurement of environmental output, namely the area of permanent grassland, delivers, on average, similar efficiency estimates as the proxy of the environmental output defined as the ratio of permanent grassland to total agricultural area as a proxy for environmental output. However, my approach is consistent with economic theory and provides more plausible results regarding estimated technology.

However, it should be noted that the results of the technical efficiency analysis show that, although the estimated efficiencies on average do not differ between the models which neglect the environmental output and those which account for this output, the individual efficiencies of farms differ, regardless of which proxy of environmental output is used.

Moreover, the paper finds that, besides significant regional heterogeneity which influences the productivity of the analysed farms, being located in less favoured areas (LFA) is negatively related to a farm's productivity.

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Appendix

A. Results of Parametric Translog Stochastic Ray Frontier

Table A1: Results of Translog stochastic ray frontier Model I and Model II A

		Model I			Model II A		
Variable	Parameter	Estimate	S.E.	P-value	Estimate	S.E.	P-value
(Intercept)	α_0	0.773	0.219	0.000	-1.312	0.411	0.001
$\ln(L)$	β_1	-0.065	0.148	0.663	-0.181	0.192	0.345
$\ln(A)$	β_2	0.190	0.102	0.063	0.471	0.150	0.002
$\ln(V)$	β_3	0.869	0.102	0.000	1.096	0.168	0.000
$\ln(K)$	β_4	0.026	0.103	0.799	0.283	0.130	0.030
θ_1	α_1	-2.058	0.248	0.000	0.871	0.337	0.010
θ_2	α_2	-	-	-	0.725	0.126	0.000
$1/2\ \ln(L)^2$	β_{11}	0.160	0.089	0.072	0.186	0.080	0.019
$\ln(L)\ln(A)$	β_{12}	0.070	0.048	0.146	0.033	0.046	0.476
$\ln(L)\ln(V)$	β_{13}	-0.124	0.049	0.011	-0.088	0.052	0.090
$\ln(L)\ln(K)$	β_{14}	0.058	0.042	0.163	0.056	0.038	0.145
$\ln(L)\theta_1$	ζ_{11}	0.041	0.099	0.682	0.131	0.078	0.095
$\ln(L)\theta_2$	ζ_{21}	-	-	-	0.027	0.039	0.486
$1/2\ \ln(A)^2$	β_{22}	-0.121	0.045	0.007	-0.098	0.042	0.019
$\ln(A)\ln(V)$	β_{23}	-0.012	0.034	0.730	-0.028	0.037	0.440
$\ln(A)\ln(K)$	β_{24}	0.046	0.027	0.090	0.031	0.026	0.234
$\ln(A)\theta_1$	ζ_{12}	0.158	0.078	0.044	0.013	0.063	0.834
$\ln(A)\theta_2$	ζ_{22}	-	-	-	-0.087	0.029	0.003
$1/2\ \ln(V)^2$	β_{33}	0.116	0.040	0.004	0.102	0.050	0.041
$\ln(V)\ln(K)$	β_{34}	-0.088	0.030	0.003	-0.097	0.031	0.001
$\ln(V)\theta_1$	ζ_{13}	0.055	0.070	0.432	-0.308	0.076	0.000
$\ln(V)\theta_2$	ζ_{23}	-	-	-	-0.188	0.032	0.000
$1/2\ \ln(K)^2$	β_4	0.060	0.033	0.063	0.034	0.029	0.248
$\ln(K)\theta_1$	ζ_{14}	-0.044	0.061	0.468	-0.137	0.050	0.007
$\ln(K)\theta_2$	ζ_{24}	-	-	-	0.005	0.024	0.832
$1/2\ \theta_1^2$	α_{11}	2.361	0.258	0.000	1.255	0.182	0.000
$\theta_1\theta_2$	α_{12}	-	-	-	-0.207	0.062	0.001
$1/2\ \theta_2^2$	α_{22}	-	-	-	0.775	0.048	0.000
REG1	ρ_{11}	0.003	0.014	0.813	0.058	0.013	0.000
REG2	ρ_{12}	-0.036	0.012	0.004	-0.029	0.011	0.011
REG4	ρ_{13}	-0.029	0.017	0.092	0.034	0.016	0.031
LFA	ρ_2	-0.058	0.011	0.000	-0.019	0.011	0.073
	σ^2	0.089	0.005	0.000	0.073	0.004	0.000
	γ	0.659	0.044	0.000	0.674	0.042	0.000
log likelihood value		173.717			421.770		
mean efficiency		0.834			0.845		

Table A2: Results of Translog stochastic ray frontier Model II B and C

		Model II B			Model II C		
Variable	Parameter	Estimate	S.E.	P-value	Estimate	S.E.	P-value
(Intercept)	α_0	1.141	0.268	0.000	0.594	0.224	0.008
$\ln(L)$	β_1	-0.076	0.152	0.616	-0.035	0.147	0.812
$\ln(A)$	β_2	0.375	0.132	0.005	0.213	0.105	0.042
$\ln(V)$	β_3	0.656	0.127	0.000	0.850	0.105	0.000
$\ln(K)$	β_4	0.001	0.101	0.991	0.069	0.102	0.499
θ_1	α_1	-1.498	0.346	0.000	-1.900	0.251	0.000
θ_2	α_2	0.272	0.135	0.044	0.550	0.174	0.002
$1/2\ \ln(L)^2$	β_{11}	0.111	0.078	0.154	0.152	0.087	0.083
$\ln(L)\ln(A)$	β_{12}	0.038	0.052	0.467	0.076	0.049	0.116
$\ln(L)\ln(V)$	β_{13}	-0.070	0.050	0.158	-0.099	0.049	0.044
$\ln(L)\ln(K)$	β_{14}	0.038	0.037	0.297	0.034	0.041	0.406
$\ln(L)\theta_1$	ζ_{11}	0.036	0.111	0.748	0.033	0.098	0.738
$\ln(L)\theta_2$	ζ_{21}	0.006	0.054	0.915	-0.038	0.082	0.647
$1/2\ \ln(A)^2$	β_{22}	-0.089	0.056	0.108	-0.063	0.047	0.183
$\ln(A)\ln(V)$	β_{23}	-0.034	0.042	0.414	-0.044	0.036	0.222
$\ln(A)\ln(K)$	β_{24}	0.018	0.029	0.524	0.037	0.027	0.176
$\ln(A)\theta_1$	ζ_{12}	0.482	0.111	0.000	0.142	0.082	0.083
$\ln(A)\theta_2$	ζ_{22}	0.296	0.042	0.000	0.068	0.052	0.191
$1/2\ \ln(V)^2$	β_{33}	0.117	0.045	0.009	0.117	0.041	0.005
$\ln(V)\ln(K)$	β_{34}	-0.046	0.029	0.107	-0.073	0.030	0.015
$\ln(V)\theta_1$	ζ_{13}	-0.253	0.102	0.013	0.062	0.073	0.397
$\ln(V)\theta_2$	ζ_{23}	-0.271	0.038	0.000	-0.175	0.059	0.003
$1/2\ \ln(K)^2$	β_4	0.050	0.028	0.079	0.051	0.032	0.112
$\ln(K)\theta_1$	ζ_{14}	-0.103	0.066	0.118	-0.069	0.061	0.254
$\ln(K)\theta_2$	ζ_{24}	-0.039	0.028	0.166	-0.004	0.046	0.925
$1/2\ \theta_1^2$	α_{11}	2.244	0.310	0.000	2.371	0.252	0.000
$\theta_1\theta_2$	α_{12}	-1.344	0.112	0.000	-0.520	0.118	0.000
$1/2\ \theta_2^2$	α_{22}	0.486	0.059	0.000	0.054	0.110	0.626
REG1	ρ_{11}	0.040	0.013	0.001	0.007	0.014	0.620
REG2	ρ_{12}	-0.035	0.011	0.001	-0.027	0.012	0.027
REG4	ρ_{13}	0.024	0.015	0.120	-0.005	0.017	0.753
LFA	ρ_2	-0.021	0.010	0.043	-0.054	0.011	0.000
	σ^2	0.069	0.004	0.000	0.075	0.006	0.000
	γ	0.685	0.039	0.000	0.567	0.064	0.000
log likelihood value		510.754			244.650		
mean efficiency		0.848			0.855		

Table A3: Results of the nonparametric estimation of the stochastic ray function Model I

Regressor	Bandwidth	Gradients					P-Value
		Min	Median	Mean	Max	Std. Dev	
$\ln(L)$	0.615	0.000 (-0.279)	0.107 (0.085)	0.113 (0.085)	0.534 (0.477)	0.055 (0.064)	< 0.001 (< 0.001)
$\ln(A)$	0.973	0.000 (-0.195)	0.126 (0.119)	0.114 (0.103)	0.347 (0.369)	0.055 (0.065)	< 0.001 (< 0.001)
$\ln(V)$	240959	0.376 (0.386)	0.762 (0.771)	0.769 (0.780)	1.021 (1.140)	0.069 (0.072)	< 0.001 (< 0.001)
$\ln(K)$	1.061	0.000 (-0.035)	0.169 (0.172)	0.163 (0.166)	0.403 (0.402)	0.041 (0.044)	< 0.001 (< 0.001)
θ	0.125	-1.672 (-1.747)	-0.912 (-0.909)	-0.879 (-0.876)	0.776 (0.776)	0.350 (0.350)	< 0.001 (< 0.001)
REG1	0.061	-0.403 (-0.471)	-0.017 (-0.020)	-0.015 (-0.017)	0.947 (0.965)	0.010 (0.094)	< 0.001 (< 0.001)
REG2	0.061	-0.292 (-0.291)	-0.033 (-0.033)	-0.035 (-0.035)	0.194 (0.211)	0.056 (0.058)	< 0.001 (< 0.001)
REG4	0.061	-0.267 (-0.460)	-0.006 (-0.004)	-0.011 (-0.014)	0.230 (0.264)	0.078 (0.095)	< 0.001 (< 0.001)
LFA	0.254	-0.260 (-0.256)	-0.022 (-0.022)	-0.022 (-0.022)	0.402 (0.411)	0.037 (0.038)	< 0.001 (< 0.001)
$R^2 = 0.940$ (0.940)							

Note: Values in parenthesis are for the monotonicity unrestricted model.

Table A4: Results of the nonparametric estimation of the stochastic ray function Model II A

Regressor	Bandwidth	Gradients					P-Value
		Min	Median	Mean	Max	Std. Dev	
$\ln(L)$	143789	0.000 (-0.216)	0.090 (0.058)	0.091 (0.051)	0.313 (0.337)	0.043 (0.055)	0.005 (0.005)
$\ln(A)$	1.344	0.000 (-0.201)	0.169 (0.170)	0.170 (0.170)	0.431 (0.470)	0.052 (0.058)	< 0.001 (< 0.001)
$\ln(V)$	0.522	0.000 (-0.232)	0.548 (0.556)	0.536 (0.543)	0.998 (1.140)	0.138 (0.143)	< 0.001 (< 0.001)
$\ln(K)$	101159	0.000 (-0.053)	0.131 (0.131)	0.131 (0.130)	0.404 (0.403)	0.041 (0.044)	< 0.001 (< 0.001)
θ_1	0.172	-4.138 (-4.321)	-0.676 (-0.673)	-0.702 (-0.704)	1.103 (0.729)	0.439 (0.433)	< 0.001 (< 0.001)
θ_2	0.173	-1.848 (-1.756)	0.179 (0.178)	0.339 (0.341)	4.001 (4.026)	0.622 (0.624)	< 0.001 (< 0.001)
REG1	0.187	-0.333 (-0.332)	0.020 (0.023)	0.018 (0.018)	0.338 (0.340)	0.061 (0.065)	< 0.001 (< 0.001)
REG2	0.187	-0.425 (-0.423)	-0.017 (-0.017)	-0.015 (-0.015)	0.200 (0.215)	0.045 (0.047)	< 0.001 (< 0.001)
REG3	0.187	-0.266 (-0.411)	0.016 (0.014)	0.011 (0.010)	0.302 (0.318)	0.069 (0.076)	< 0.001 (< 0.001)
LFA	0.814	-0.024 (-0.026)	-0.001 (-0.001)	-0.001 (-0.001)	0.041 (0.042)	0.006 (0.006)	< 0.001 (< 0.001)
$R^2 = 0.943$ (0.942)							

Note: Values in parenthesis are for the monotonicity unrestricted model.

Table A5: Results of the nonparametric estimation of the stochastic ray function Model II B

Regressor	Bandwidth	Gradients					P-Value
		Min	Median	Mean	Max	Std. Dev	
$\ln(L)$	40481	0.000 (-0.325)	0.087 (0.054)	0.096 (0.052)	0.577 (0.514)	0.050 (0.060)	0.005 (0.005)
$\ln(A)$	0.988	0.000 (-0.159)	0.324 (0.326)	0.338 (0.345)	0.923 (1.200)	0.120 (0.139)	< 0.001 (< 0.001)
$\ln(V)$	2704933	0.000 (-0.230)	0.569 (0.575)	0.546 (0.549)	0.838 (0.878)	0.115 (0.126)	< 0.001 (< 0.001)
$\ln(K)$	1.645	0.000 (-0.126)	0.138 (0.137)	0.135 (0.133)	0.403 (0.403)	0.035 (0.040)	< 0.001 (< 0.001)
θ_1	0.081	-3.711 (-3.699)	-1.358 (-1.358)	-1.478 (-1.484)	0.661 (0.533)	0.694 (0.697)	< 0.001 (< 0.001)
θ_2	0.196	-2.013 (-2.052)	-0.330 (-0.328)	-0.399 (-0.401)	0.737 (0.537)	0.360 (0.362)	< 0.001 (< 0.001)
REG1	0.070	-0.237 (-0.248)	0.018 (0.014)	0.016 (0.014)	0.838 (0.519)	0.064 (0.069)	< 0.001 (< 0.001)
REG2	0.070	-0.425 (-0.330)	-0.017 (-0.005)	-0.015 (-0.008)	0.200 (0.152)	0.045 (0.041)	< 0.001 (< 0.001)
REG4	0.070	-0.266 (-0.387)	0.015 (0.011)	0.011 (0.014)	0.302 (0.315)	0.069 (0.069)	< 0.001 (< 0.001)
LFA	0.536	-0.116 (-0.115)	-0.004 (-0.005)	-0.004 (-0.004)	0.119 (0.116)	0.016 (0.017)	< 0.001 (< 0.001)
$R^2 = 0.958$ (0.958)							

Note: Values in parenthesis are for the monotonicity unrestricted model.

Table A6: Results of the nonparametric estimation of the stochastic ray function Model II C

Regressor	Bandwidth	Gradients					P-Value
		Min	Median	Mean	Max	Std. Dev	
ln(L)	0.513	0.000 (-0.312)	0.106 (0.084)	0.114 (0.085)	0.506 (0.481)	0.059 (0.059)	< 0.001 (< 0.001)
ln(A)	3.862	0.000 (-0.167)	0.140 (0.142)	0.128 (0.128)	0.678 (0.670)	0.064 (0.064)	< 0.001 (< 0.001)
ln(V)	154386	0.343 (0.356)	0.724 (0.729)	0.739 (0.747)	1.019 (1.125)	0.077 (0.077)	< 0.001 (< 0.001)
ln(K)	0.924	0.000 (-0.127)	0.175 (0.175)	0.171 (0.169)	0.548 (0.546)	0.041 (0.041)	< 0.001 (< 0.001)
θ_1	0.130	-1.992 (-1.953)	-0.944 (-0.944)	-0.905 (-0.906)	0.756 (0.735)	0.357 (0.357)	< 0.001 (< 0.001)
θ_2	0.431	-0.857 (-0.835)	-0.189 (-0.187)	-0.186 (-0.180)	0.318 (0.348)	0.145 (0.145)	< 0.001 (< 0.001)
REG1	0.070	-0.354 (-0.489)	-0.010 (-0.008)	-0.003 (-0.006)	0.838 (0.895)	0.092 (0.100)	< 0.001 (< 0.001)
REG2	0.070	-0.220 (-0.225)	-0.021 (-0.021)	-0.023 (-0.023)	0.219 (0.223)	0.058 (0.058)	< 0.001 (< 0.001)
REG4	0.070	-0.291 (-0.371)	-0.004 (-0.004)	0.017 (0.017)	0.639 (0.638)	0.126 (0.132)	< 0.001 (< 0.001)
LFA	0.319	-0.179 (-0.180)	-0.015 (-0.016)	-0.015 (-0.016)	0.317 (0.318)	0.034 (0.031)	< 0.001 (< 0.001)
$R^2 = 0.945$ (0.945)							

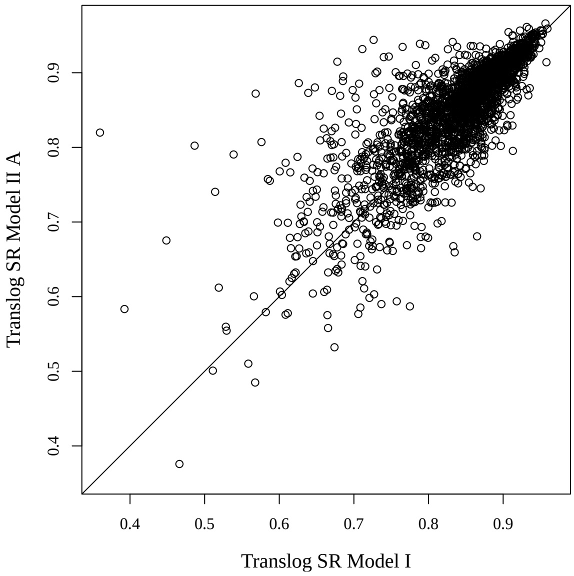
Note: Values in parenthesis are for the monotonicity unrestricted model.

Table A7: Results of semiparametric stochastic ray (frontier) production function

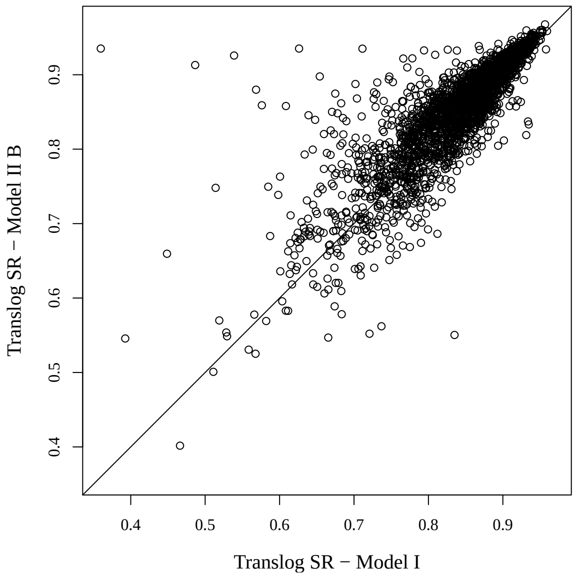
	Model I			Model II		
Regressor	Estimate	S.E.	P-value	Estimate	S.E.	P-value
(Intercept)	0.165	0.011	0.000	0.124	0.010	0.000
σ^2	0.073	0.004	0.000	0.049	0.003	0.000
γ	0.630	0.045	0.000	0.623	0.047	0.000
log likelihood value	363.401			842.913		
mean efficiency	0.850			0.875		

Table A8: Results of semiparametric stochastic ray (frontier) production function

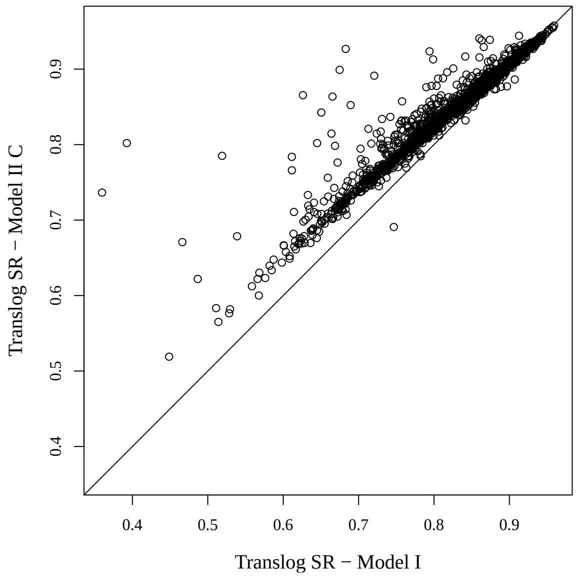
	Model II			Model II		
Regressor	Estimate	S.E.	P-value	Estimate	S.E.	P-value
(Intercept)	0.119	0.011	0.000	0.134	0.015	0.000
σ^2	0.047	0.003	0.000	0.060	0.004	0.000
γ	0.574	0.056	0.000	0.528	0.068	0.000
log likelihood value	820.926			473.450		
mean efficiency	0.888			0.873		



(a) Translog Model I vs. Translog Model II A

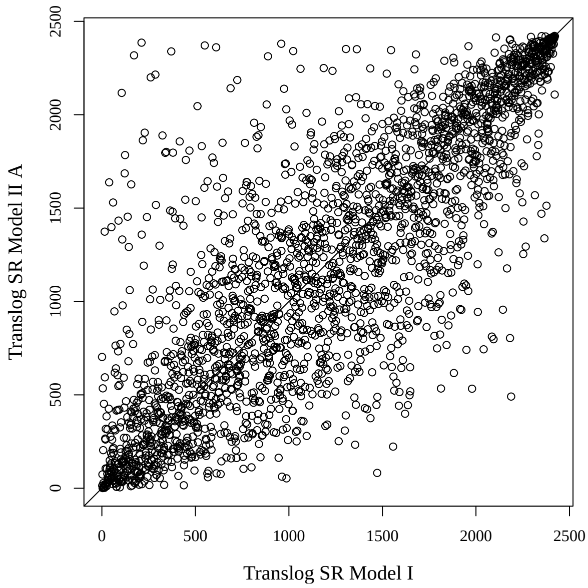


(b) Translog Model I vs. Translog Model II B

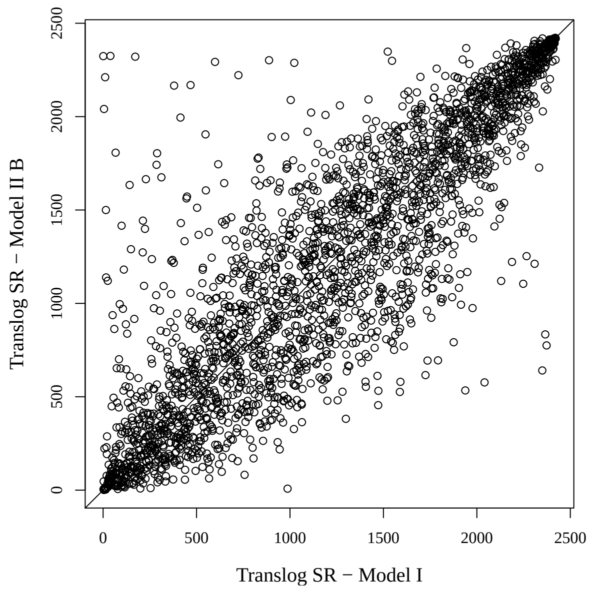


(c) Translog Model I vs. Translog Model II C

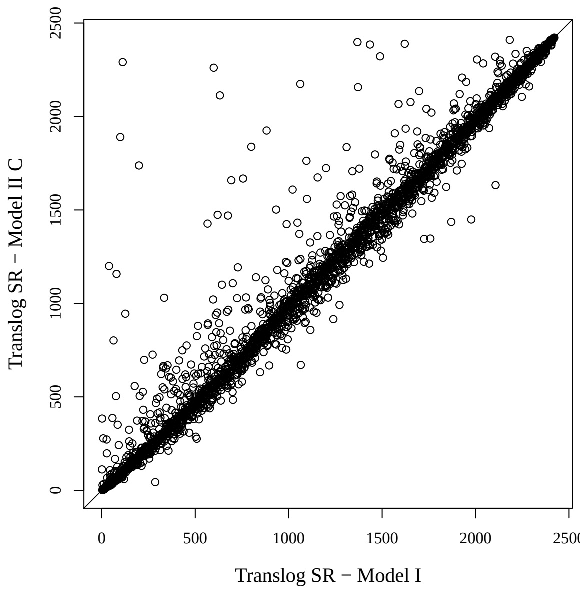
Figure A1: Comparison of technical efficiencies obtained from the Translog stochastic ray frontiers for Model I and Model II A (A1a), Model I and Model II B (A1b) and Model I and Model II C (A1c)



(a) Translog Model I vs. Translog Model II A

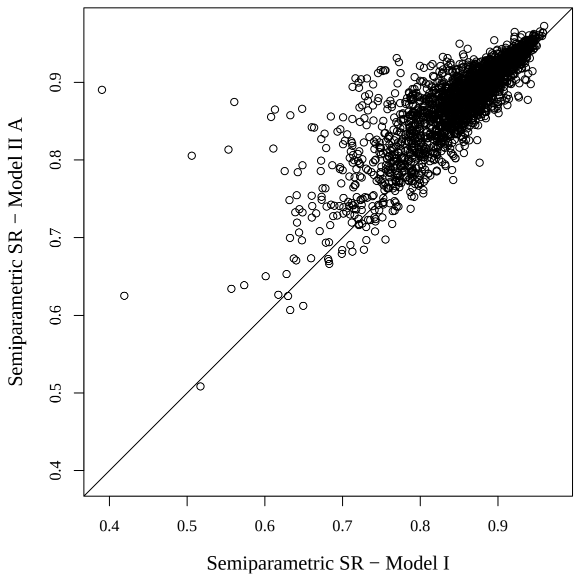


(b) Translog Model I vs. Translog Model II B

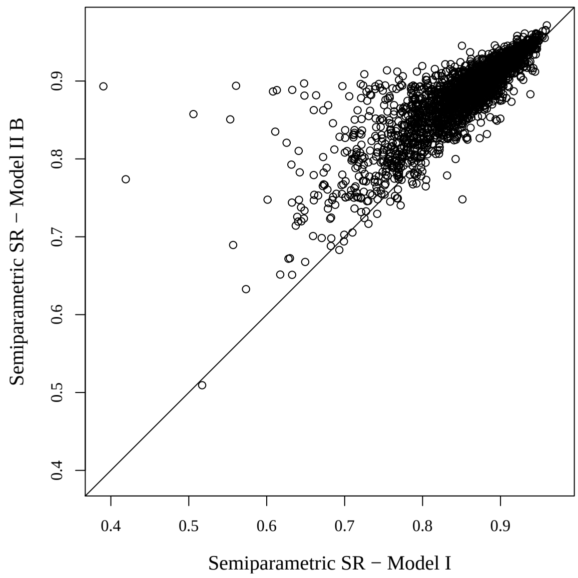


(c) Translog Model I vs. Translog Model II C

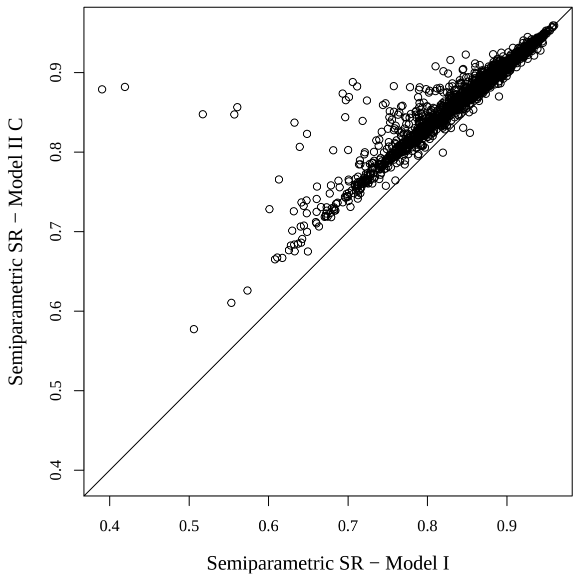
Figure A2: Comparison of rankings based on the technical efficiency scores obtained from the Translog stochastic ray frontiers for Model I and Model II A (A2a), Model I and Model II B (A2b) and Model I and Model II C (A2c)



(a) Semipar. SR Model I vs. Model II A

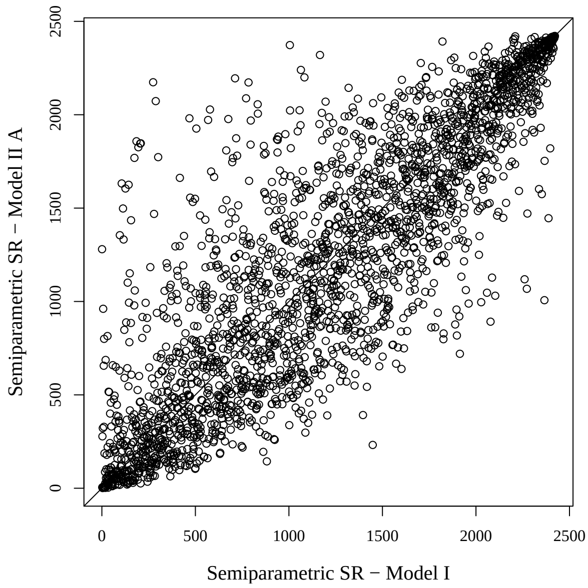


(b) Semipar. SR Model I vs. Model II B

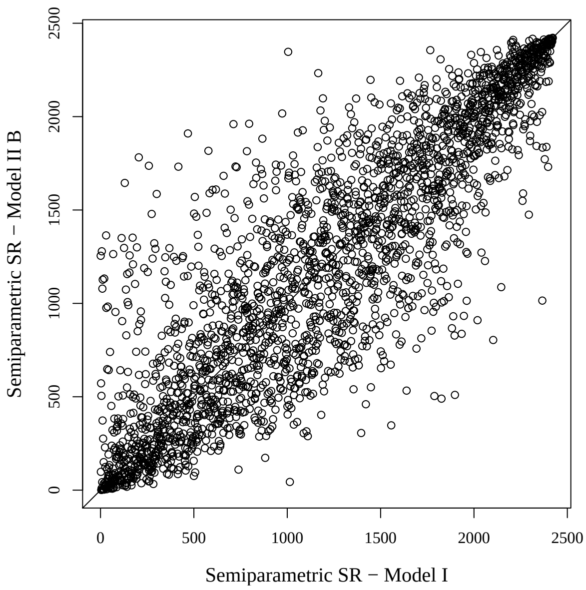


(c) Semipar. SR Model I vs. Model II C

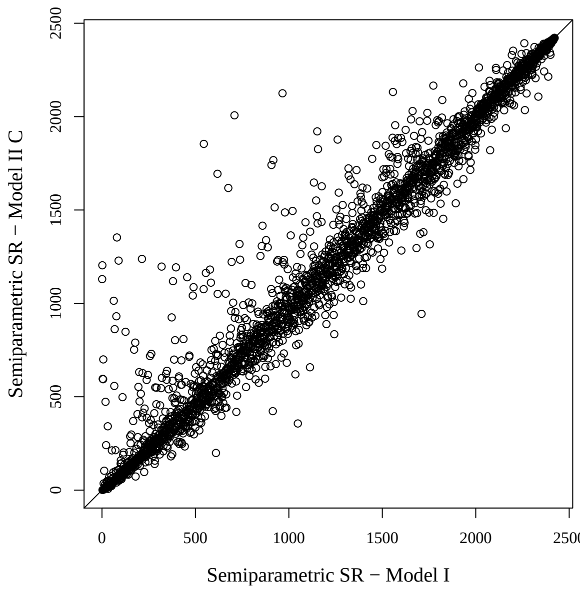
Figure A3: Comparison of technical efficiencies obtained from the semiparametric stochastic ray frontiers for Model I and Model II A (A3a), Model I and Model II B (A3b) and Model I and Model II C (A3c)



(a) Semipar. SR Model I vs. Model II A

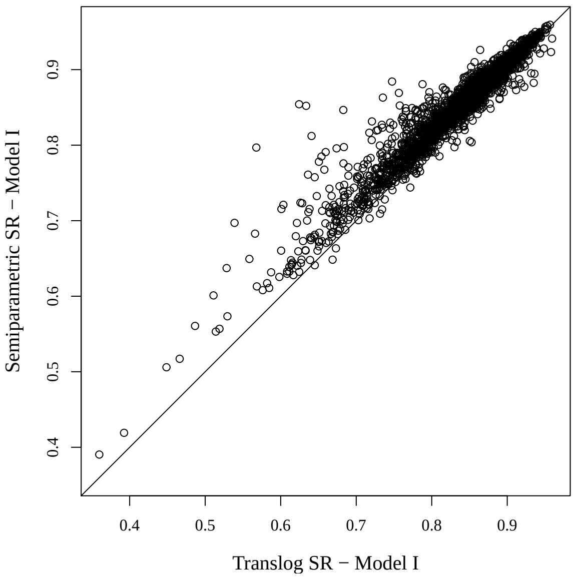


(b) Semipar. SR Model I vs. Model II B

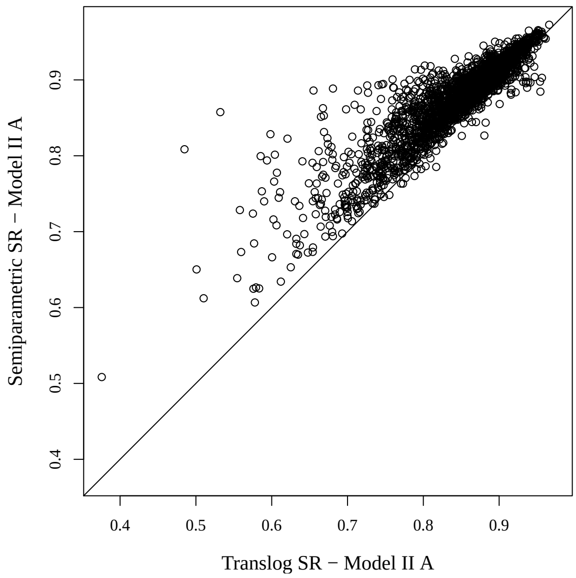


(c) Semipar. SR Model I vs. Model II C

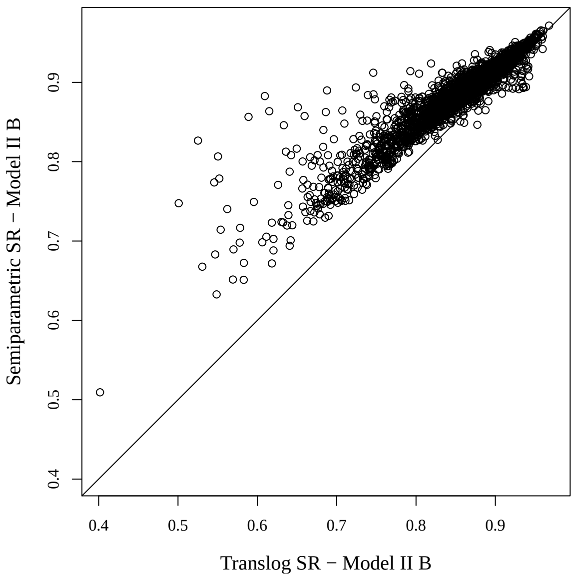
Figure A4: Comparison of rankings based on the technical efficiency scores obtained from the semiparametric ray frontiers for Model I and Model II A (A4a), Model I and Model II B (A4b) and Model I and Model II C (A4c)



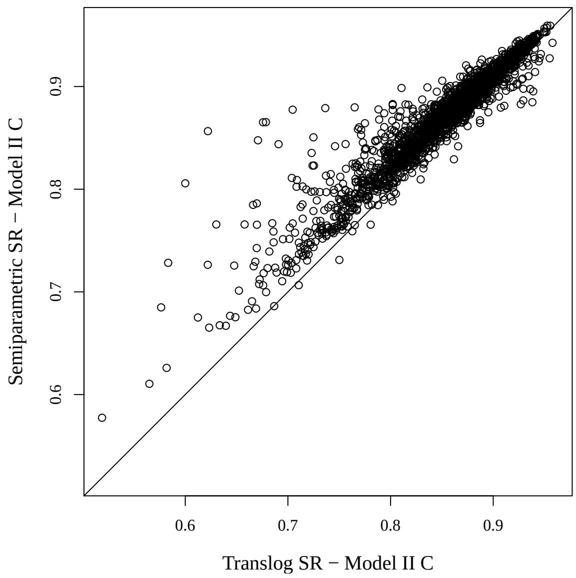
(a) Translog SR and Semipar. SR Model I



(b) Translog SR and Semipar. SR Model II A

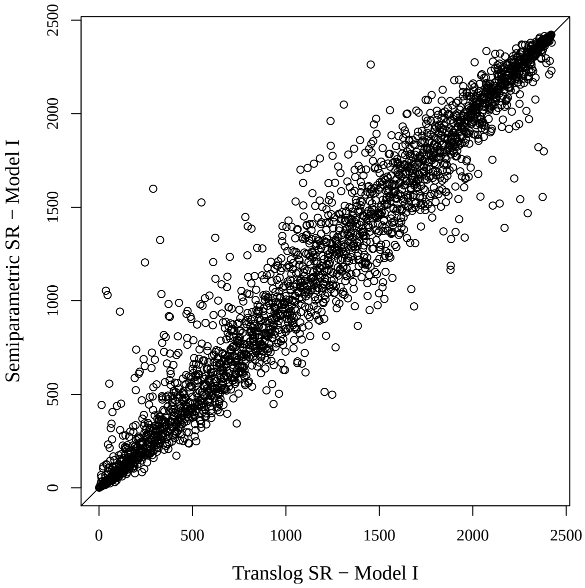


(c) Translog SR Model I Model II B

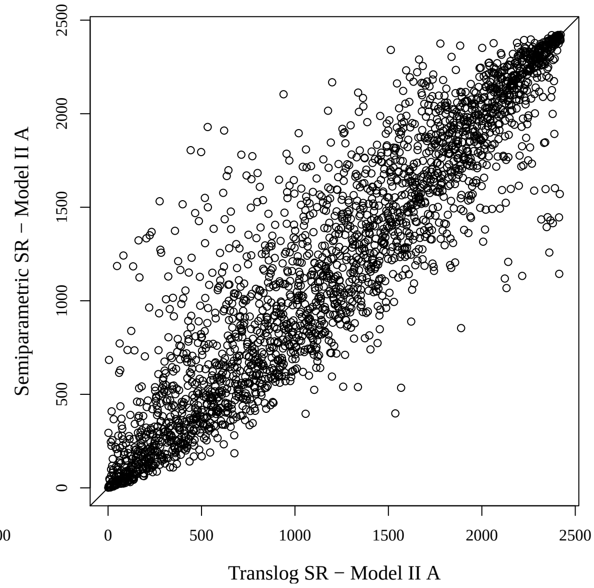


(d) Semipar. SR Model I Model II C

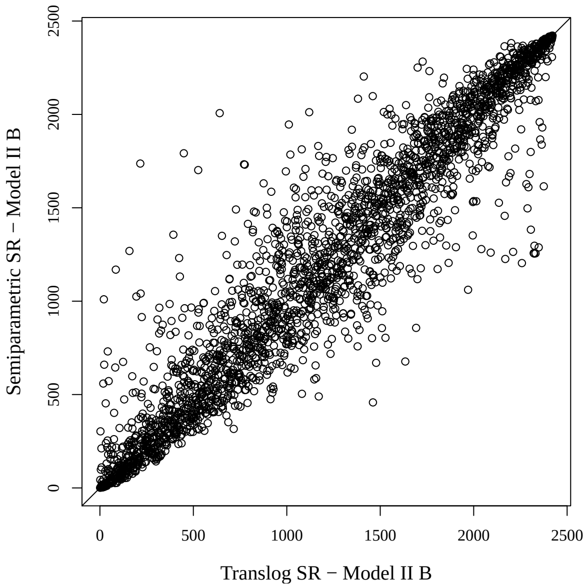
Figure A5: Comparison of technical efficiencies obtained from the Translog and semiparametric stochastic ray frontiers for Model I (A5a), Model II A (A5b), Model II B (A5c) and Model II C (A5d)



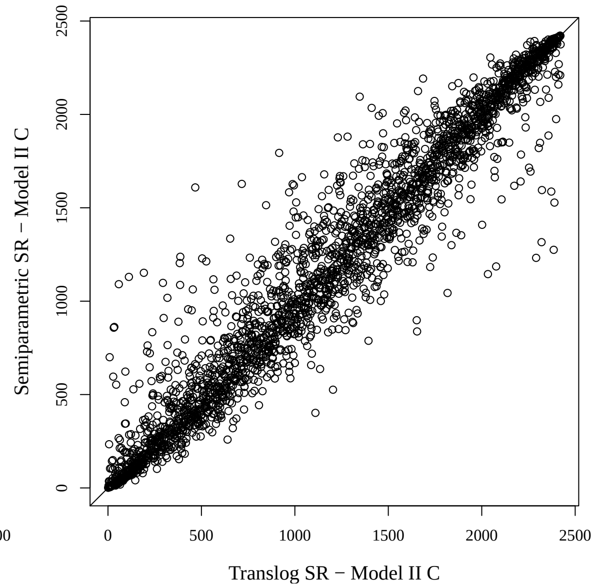
(a) Translog SR and Semipar. SR Model I



(b) Translog SR and Semipar. SR Model II A



(c) Translog SR and Semipar. SR Model II B



(d) Translog SR and Semipar. SR Model II C

Figure A6: Comparison of rankings based on the technical efficiency scores obtained from the Translog and semiparametric stochastic ray frontiers for Model I (A6a), Model II A (A6b), Model II B (A6c) and Model II C (A6d)

B. Derivation of output distance elasticities of stochastic ray function

Output distance function can be expressed in terms of the ray production function:

$$D_o = \frac{\|y\|}{f(x,\theta)} \tag{14}$$

$$\ln D_o = \ln \|y\| - \ln f(x,\theta) \tag{15}$$

B.1. 2 outputs case:

The Euclidean norm ($\|y\|$) of the output vector y in to output case is given by:

$$\|y\| = \sqrt{y_1^2 + y_2^2} \tag{16}$$

The polar coordinate angle vector ($\theta \in [0, \pi/2]$) can be expressed as:

$$\theta = \arccos\left(\frac{y_1}{\sqrt{y_1^2 + y_2^2}}\right) \tag{17}$$

The output distance elasticity with respect to output 1 ($\frac{\partial \ln D_o}{\partial \ln y_1}$) is given by:

$$\frac{\partial \ln D_o}{\partial \ln y_1} = \frac{\partial \ln \|y\|}{\partial y_1} \cdot \frac{\partial y}{\partial \ln y_1} - \frac{\partial \ln f(x,\theta)}{\partial \theta} \cdot \frac{\partial \theta}{\partial y_1} \cdot \frac{\partial y_1}{\partial \ln y_1} \tag{18}$$

The output distance elasticity with respect to output 2 ($\frac{\partial \ln D_o}{\partial \ln y_2}$) is given by:

$$\frac{\partial \ln D_o}{\partial \ln y_2} = \frac{\partial \ln \|y\|}{\partial y_2} \cdot \frac{\partial y}{\partial \ln y_2} - \frac{\partial \ln f(x,\theta)}{\partial \theta} \cdot \frac{\partial \theta}{\partial y_2} \cdot \frac{\partial y_2}{\partial \ln y_2} \tag{19}$$

$$\frac{\partial \ln \|y\|}{\partial y_1} = \frac{\partial \ln \sqrt{y_1^2 + y_2^2}}{\partial y_1} = \frac{1}{\sqrt{y_1^2 + y_2^2}} \cdot \frac{1}{2\sqrt{y_1^2 + y_2^2}} \cdot 2y_1 = \frac{y_1}{\|y\|^2} \tag{20}$$

The partial derivative of the Euclidean norm ($\|y\|$) with respect to $i-th$ output is equal:

$$\frac{\partial \ln \|y\|}{\partial y_i} = \frac{\partial \ln \sqrt{y_1^2 + y_2^2}}{\partial y_i} = \frac{1}{\sqrt{y_1^2 + y_2^2}} \cdot \frac{1}{2\sqrt{y_1^2 + y_2^2}} \cdot 2y_i = \frac{y_i}{\|y\|^2} \tag{21}$$

The partial derivative of the $i-th$ output with respect to the log of the $i-th$ output is equal:

$$\frac{\partial y_i}{\partial \ln y_i} = \frac{1}{\frac{\partial \ln y_i}{\partial y_i}} = \frac{1}{\frac{1}{y_i}} = y_i \tag{22}$$

The partial derivative of the polar coordinate angle vector with respect to $1st$ output is equal:

$$\frac{\partial \theta}{\partial y_1} = \frac{\partial \arccos\left(\frac{y_1}{\sqrt{y_1^2+y_2^2}}\right)}{\partial y_1} = \frac{-1}{\sqrt{1 - \frac{y_1^2}{y_1^2+y_2^2}}} \cdot \left(\frac{1}{\sqrt{y_1^2 + y_2^2}} - \frac{y_1^2}{\sqrt{y_1^2 + y_2^2}\,(y_1^2 + y_2^2)}\right) \tag{23}$$

$$\frac{\partial \theta}{\partial y_1} = \frac{-1}{\sqrt{\frac{y_1^2+y_2^2-y_1^2}{y_1^2+y_2^2}}} \cdot \left(\frac{1}{\sqrt{y_1^2 + y_2^2}} - \frac{y_1^2}{\sqrt{y_1^2 + y_2^2}\,(y_1^2 + y_2^2)}\right) \tag{24}$$

$$\frac{\partial\theta}{\partial y_1} = -\frac{\sqrt{y_1^2+y_2^2}}{y_2}\cdot\left(\frac{1}{\sqrt{y_1^2+y_2^2}} - \frac{y_1^2}{\sqrt{y_1^2+y_2^2}\,(y_1^2+y_2^2)}\right) \tag{25}$$

$$\frac{\partial\theta}{\partial y_1} = -\frac{1}{y_2}\cdot\left(\frac{y_1^2+y_2^2-y_1^2}{(y_1^2+y_2^2)}\right) = -\frac{1}{y_2}\cdot\left(\frac{y_2^2}{(y_1^2+y_2^2)}\right) = \frac{-y_2}{(y_1^2+y_2^2)} = \frac{-y_2}{\|y\|^2} \tag{26}$$

The partial derivative of the polar coordinate angle vector with respect to $2nd$ output is equal:

$$\frac{\partial\theta}{\partial y_2} = \frac{\partial \arccos\left(\frac{y_1}{\sqrt{y_1^2+y_2^2}}\right)}{\partial y_2} = -\frac{\sqrt{y_1^2+y_2^2}}{y_2}\cdot\left(0.5\cdot\frac{y_1}{\sqrt{y_1^2+y_2^2}\cdot(y_1^2+y_2^2)}\cdot 2y_2\right) \tag{27}$$

$$\frac{\partial\theta}{\partial y_2} = \frac{-y_1}{(y_1^2+y_2^2)} = \frac{-y_1}{\|y\|^2} \tag{28}$$

The partial derivative of the stochastic ray production function of the Translog functional (7) with respect to the polar coordinate angle vector (θ) is given by:

$$\frac{\partial \ln f(x,\theta)}{\partial\theta} = \alpha_1 + \alpha_{11}\theta + \sum_k \zeta \ln x_k \tag{29}$$

Finally, the output distance elasticity with respect to output $i-th$ ($\frac{\partial \ln D_o}{\partial \ln y_i}$) is equal:

$$\frac{\partial \ln D_o}{\partial \ln y_i} = \frac{y_{3-i}}{\|y\|^2}\cdot y_2 - (\alpha_1 + \alpha_{11}\theta + \sum_k \zeta \ln x_k)\cdot\frac{-y_i y_{3-i}}{\|y\|^2} \tag{30}$$

B.2. 3 outputs case:

$$\|y\| = \sqrt{y_1^2+y_2^2+y_3^2} \tag{31}$$

$$\theta_1 = \arccos\left(\frac{y_1}{\sqrt{y_1^2+y_2^2+y_3^2}}\right) \tag{32}$$

$$\theta_2 = \arccos\left(\frac{y_2}{\sqrt{y_1^2+y_2^2+y_3^2}}\right)\cdot\sin(\theta_1) \tag{33}$$

$$\frac{\partial \ln D_o}{\partial \ln y_1} = \frac{\partial \ln\|y\|}{\partial y_1}\cdot\frac{\partial y}{\partial \ln y_1} - \frac{\partial \ln f(x,\theta_1,\theta_2)}{\partial\theta_1}\cdot\frac{\partial\theta_1}{\partial y_1}\cdot\frac{\partial y_1}{\partial \ln y_1} \tag{34}$$

$$-\frac{\partial \ln f(x,\theta_1,\theta_2)}{\partial\theta_2}\cdot\frac{\partial\theta_2}{\partial y_1}_{|\theta_1=const.}\cdot\frac{\partial y_1}{\partial \ln y_1} - \frac{\partial \ln f(x,\theta_1,\theta_2)}{\partial\theta_2}\cdot\frac{\partial\theta_2}{\partial\theta_1}\cdot\frac{\partial\theta_1}{\partial y_1}\cdot\frac{\partial y_1}{\partial \ln y_1} \tag{35}$$

$\frac{\partial \ln\|y\|}{\partial y_i}$:

$$\frac{\partial \ln\|y\|}{\partial y_i} = \frac{\partial \ln\sqrt{y_1^2+y_2^2+y_3^2}}{\partial y_i} = \frac{1}{\sqrt{y_1^2+y_2^2+y_3^2}}\cdot\frac{1}{2\sqrt{y_1^2+y_2^2+y_3^2}}\cdot 2y_i = \frac{y_i}{\|y\|^2} \tag{36}$$

$\frac{\partial \ln y_i}{\partial \ln y_i}$:

$$\frac{\partial \ln y_i}{\partial \ln y_i} = \frac{1}{\frac{\partial \ln y_i}{\partial \ln y_i}} = \frac{1}{\frac{1}{y_I}} = y_i \tag{37}$$

$\frac{\partial\theta_2}{\partial\theta_1}$:

$$\frac{\partial\theta_2}{\partial\theta_1} = \frac{\partial \arccos\left(\frac{y_2}{\sqrt{y_1^2+y_2^2+y_3^2}}\right)\cdot\sin(\theta_1)}{\partial\theta_1} = \arccos\left(\frac{y_2}{\sqrt{y_1^2+y_2^2+y_3^2}}\right)\cdot\cos(\theta_1) \tag{38}$$

$$\frac{\partial\theta_2}{\partial\theta_1} = \arccos\left(\frac{y_2}{\sqrt{y_1^2+y_2^2+y_3^2}}\right)\cdot\cos\left(\arccos\left(\frac{y_1}{\sqrt{y_1^2+y_2^2+y_3^2}}\right)\right) \tag{39}$$

$$\frac{\partial\theta_2}{\partial\theta_1} = \arccos\left(\frac{y_2}{\sqrt{y_1^2+y_2^2+y_3^2}}\right)\cdot\left(\frac{y_1}{\sqrt{y_1^2+y_2^2+y_3^2}}\right) = \left(\frac{y_1}{\|y\|}\right)\cdot\arccos\left(\frac{y_2}{\|y\|}\right) \tag{40}$$

$\frac{\partial\theta_2}{\partial y_1}$:

$$\frac{\partial\theta_2}{\partial y_1}_{|\theta_1=const.} = \frac{\partial \arccos\left(\frac{y_2}{\sqrt{y_1^2+y_2^2+y_3^2}}\right)\cdot\sin(\theta_1)}{\partial y_1} \tag{41}$$

$$\frac{\partial\theta_2}{\partial y_1}_{|\theta_1=const.} = \frac{-1}{\sqrt{1-\frac{y_2^2}{y_1^2+y_2^2+y_3^2}}}\cdot\left(-0.5\cdot\frac{y_2}{\sqrt{y_1^2+y_2^2+y_3^2}\cdot(y_1^2+y_2^2+y_3^2)}\cdot 2y_1\right)\cdot\sin(\theta_1) \tag{42}$$

$$\frac{\partial\theta_2}{\partial y_1}_{|\theta_1=const.} = \frac{-1}{\sqrt{\frac{y_1^2+y_2^2+y_3^2-y_2^2}{y_1^2+y_2^2+y_3^2}}}\cdot\left(\frac{-y_1y_2}{\sqrt{y_1^2+y_2^2+y_3^2}\cdot(y_1^2+y_2^2+y_3^2)}\right)\cdot\sin(\theta_1) \tag{43}$$

$$\frac{\partial\theta_2}{\partial y_1}_{|\theta_1=const.} = -\frac{\sqrt{y_1^2+y_2^2+y_3^2}}{\sqrt{y_1^2+y_3^2}}\cdot\left(\frac{-y_1y_2}{\sqrt{y_1^2+y_2^2+y_3^2}\cdot(y_1^2+y_2^2+y_3^2)}\right)\cdot\sin(\theta_1) \tag{44}$$

$$\frac{\partial\theta_2}{\partial y_1}_{|\theta_1=const.} = \frac{y_1y_2}{\sqrt{y_1^2+y_3^2}\cdot(y_1^2+y_2^2+y_3^2)}\cdot\sin(\theta_1) = \frac{y_1y_2}{\sqrt{y_1^2+y_3^2}\cdot\|y\|^2}\cdot\sin(\theta_1) \tag{45}$$

$\frac{\partial\theta_2}{\partial y_2}$:

$$\frac{\partial\theta_2}{\partial y_2}_{|\theta_1=const.} = \frac{\partial \arccos\left(\frac{y_2}{\sqrt{y_1^2+y_2^2+y_3^2}}\right)\cdot\sin(\theta_1)}{\partial y_2} \tag{46}$$

$$\frac{\partial\theta_2}{\partial y_2}_{|\theta_1=const.} = \frac{-1}{\sqrt{1-\frac{y_2^2}{y_1^2+y_2^2+y_3^2}}}\cdot\left(\frac{1}{\sqrt{y_1^2+y_2^2+y_3^2}}-0.5\cdot\frac{y_2}{\sqrt{y_1^2+y_2^2+y_3^2}\cdot(y_1^2+y_2^2+y_3^2)}\cdot 2y_2\right)\cdot\sin(\theta_1) \tag{47}$$

$$\frac{\partial\theta_2}{\partial y_2}_{|\theta_1=const.} = \frac{-1}{\sqrt{\frac{y_1^2+y_2^2+y_3^2-y_2^2}{y_1^2+y_2^2+y_3^2}}}\cdot\left(\frac{1}{\sqrt{y_1^2+y_2^2+y_3^2}}-\frac{y_2^2}{\sqrt{y_1^2+y_2^2+y_3^2}\cdot(y_1^2+y_2^2+y_3^2)}\right)\cdot\sin(\theta_1) \tag{48}$$

$$\frac{\partial\theta_2}{\partial y_2}_{|\theta_1=const.} = \frac{1}{\sqrt{y_1^2+y_3^2}}\cdot\left(\frac{y_1^2+y_2^2+y_3^2-y_2^2}{y_1^2+y_2^2+y_3^2}\right)\cdot\sin(\theta_1) \tag{49}$$

$$\frac{\partial\theta_2}{\partial y_2}_{|\theta_1=const.} = \left(\frac{y_1^2+y_3^2}{\sqrt{y_1^2+y_3^2}\cdot(y_1^2+y_2^2+y_3^2)}\right)\cdot\sin(\theta_1) = \left(\frac{\left(\sqrt{y_1^2+y_3^2}\right)^2}{\sqrt{y_1^2+y_3^2}\cdot(y_1^2+y_2^2+y_3^2)}\right)\cdot\sin(\theta_1) \tag{50}$$

$$\frac{\partial \theta_2}{\partial y_2}_{|\theta_1=const.} = \left(\frac{\sqrt{y_1^2+y_3^2}}{y_1^2+y_2^2+y_3^2} \right) \cdot \sin(\theta_1) \tag{51}$$

$$\frac{\partial \theta_2}{\partial y_2}_{|\theta_1=const.} = \frac{\sqrt{y_1^2+y_3^2}}{\|y\|^2} \cdot \sin(\theta_1) \tag{52}$$

$\frac{\partial \theta_2}{\partial y_3}$:

$$\frac{\partial \theta_2}{\partial y_3}_{|\theta_1=const.} = \frac{\partial \arccos\left(\frac{y_2}{\sqrt{y_1^2+y_2^2+y_3^2}} \right) \cdot \sin(\theta_1)}{\partial y_3} \tag{53}$$

$$\frac{\partial \theta_2}{\partial y_3}_{|\theta_1=const.} = \frac{-1}{\sqrt{1-\frac{y_2^2}{y_1^2+y_2^2+y_3^2}}} \cdot \left(-0.5 \cdot \frac{y_2}{\sqrt{y_1^2+y_2^2+y_3^2} \cdot (y_1^2+y_2^2+y_3^2)} \cdot 2y_3 \right) \cdot \sin(\theta_1) \tag{54}$$

$$\frac{\partial \theta_2}{\partial y_3}_{|\theta_1=const.} = \frac{-1}{\sqrt{\frac{y_1^2+y_2^2+y_3^2-y_2^2}{y_1^2+y_2^2+y_3^2}}} \cdot \left(\frac{-y_2 y_3}{\sqrt{y_1^2+y_2^2+y_3^2} \cdot (y_1^2+y_2^2+y_3^2)} \right) \cdot \sin(\theta_1) \tag{55}$$

$$\frac{\partial \theta_2}{\partial y_3}_{|\theta_1=const.} = -\frac{\sqrt{y_1^2+y_2^2+y_3^2}}{\sqrt{y_1^2+y_3^2}} \cdot \left(\frac{-y_2 y_3}{\sqrt{y_1^2+y_2^2+y_3^2} \cdot (y_1^2+y_2^2+y_3^2)} \right) \cdot \sin(\theta_1) \tag{56}$$

$$\frac{\partial \theta_2}{\partial y_3}_{|\theta_1=const.} = \frac{y_2 y_3}{\sqrt{y_1^2+y_3^2} \cdot (y_1^2+y_2^2+y_3^2)} \cdot \sin(\theta_1) = \frac{y_2 y_3}{\sqrt{y_1^2+y_3^2} \cdot \|y\|^2} \cdot \sin(\theta_1) \tag{57}$$

$\frac{\partial \theta_1}{\partial y_1}$:

$$\frac{\partial \theta_1}{\partial y_1} = \frac{\partial \arccos\left(\frac{y_1}{\sqrt{y_1^2+y_2^2+y_3^2}} \right)}{\partial y_1} \tag{58}$$

$$\frac{\partial \theta_1}{\partial y_1} = \frac{-1}{\sqrt{1-\frac{y_1^2}{y_1^2+y_2^2+y_3^2}}} \cdot \left(\frac{1}{\sqrt{y_1^2+y_2^2+y_3^2}} - \frac{y_1^2}{\sqrt{y_1^2+y_2^2+y_3^2}\,(y_1^2+y_2^2+y_3^2)} \right) \tag{59}$$

$$\frac{\partial \theta_1}{\partial y_1} = \frac{-1}{\sqrt{\frac{y_1^2+y_2^2+y_3^2-y_1^2}{y_1^2+y_2^2+y_3^2}}} \cdot \left(\frac{1}{\sqrt{y_1^2+y_2^2+y_3^2}} - \frac{y_1^2}{\sqrt{y_1^2+y_2^2+y_3^2}\,(y_1^2+y_2^2+y_3^2)} \right) \tag{60}$$

$$\frac{\partial \theta_1}{\partial y_1} = -\frac{\sqrt{y_1^2+y_2^2+y_3^2}}{\sqrt{y_2^2+y_3^2}} \cdot \left(\frac{1}{\sqrt{y_1^2+y_2^2+y_3^2}} - \frac{y_1^2}{\sqrt{y_1^2+y_2^2+y_3^2}\,(y_1^2+y_2^2+y_3^2)} \right) \tag{61}$$

$$\frac{\partial \theta_1}{\partial y_1} = -\frac{1}{\sqrt{y_{12}^2+y_3^2}} \cdot \left(\frac{y_1^2+y_2^2+y_3^2-y_1^2}{(y_1^2+y_2^2+y_3^2)} \right) \tag{62}$$

$$\frac{\partial \theta_1}{\partial y_1} = -\frac{1}{\sqrt{y_2^2+y_3^2}} \cdot \left(\frac{\left(\sqrt{y_2^2+y_3^2}\right)^2}{(y_1^2+y_2^2+y_3^2)} \right) = \frac{-\sqrt{y_2^2+y_3^2}}{(y_1^2+y_2^2+y_3^2)} = \frac{-\sqrt{y_2^2+y_3^2}}{\|y\|^2} \tag{63}$$

$\frac{\partial \theta_1}{\partial y_2}$:

$$\frac{\partial \theta_1}{\partial y_2} = \frac{\partial \arccos\left(\frac{y_1}{\sqrt{y_1^2+y_2^2+y_3^2}} \right)}{\partial y_2} \tag{64}$$

$$\frac{\partial \theta_1}{\partial y_2} = \frac{-1}{\sqrt{1-\frac{y_1^2}{y_1^2+y_2^2+y_3^2}}} \left(-0.5 \cdot \frac{y_1}{\sqrt{y_1^2+y_2^2+y_3^2}\,(y_1^2+y_2^2+y_3^2)} \cdot 2y_2 \right) \tag{65}$$

$$\frac{\partial\theta_1}{\partial y_2} = -\frac{\sqrt{y_1^2+y_2^2+y_3^2}}{\sqrt{y_2^2+y_3^2}} \cdot \left(\frac{-y_1 y_2}{\sqrt{y_1^2+y_2^2+y_3^2}\,(y_1^2+y_2^2+y_3^2)}\right) \tag{66}$$

$$\frac{\partial\theta_1}{\partial y_2} = \frac{y_1 y_2}{\sqrt{y_2^2+y_3^2}\,(y_1^2+y_2^2+y_3^2)} = \frac{y_1 y_2}{\sqrt{y_2^2+y_3^2}\cdot\|y\|^2} \tag{67}$$

$\frac{\partial\theta_1}{\partial y_3}$:

$$\frac{\partial\theta_1}{\partial y_3} = \frac{\partial \arccos\left(\frac{y_1}{\sqrt{y_1^2+y_2^2+y_3^2}}\right)}{\partial y_3} = \frac{y_1 y_3}{\sqrt{y_2^2+y_3^2}\cdot\|y\|^2} \tag{68}$$

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