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Standards vs. labels with imperfect competition and asymmetric information

Author: Kenneth Baltzer

Institute of Food and Resource Economics

University of Copenhagen

Rolighedsvej 25

DK 1958 Frederiksberg DENMARK

www.foi.life.ku.dk

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Kenneth Baltzer

Institute of Food and Resource Economics, University of Copenhagen

Rolighedsvej 25, 1958 Frederiksberg, Denmark

Ph.: +45 35 33 68 52

E-mail: kb@foi.dk

Abstract

I demonstrate that providing information about product quality is not necessarily the best way to address asymmetric information problems when markets are imperfectly competitive. In a vertical differentiation model I show that a Minimum Quality Standard, which retains asymmetric information, generates more welfare than a label, which provides full information.

Keywords: minimum quality standard; label; asymmetric information; vertical differentiation; product quality

JEL classifications: L13, L15, L51

1 Introduction

The purpose of this paper is to demonstrate that providing information about product quality is not necessarily the best way to address asymmetric information problems when the market is imperfectly competitive.

Consider a market for a product of potentially varying levels of quality. Suppose the firms producing the product have control over the quality of their product, but consumers are unable to observe the level of quality (for instance, think of microbiological contaminants in food, chemical residues in toys or safety characteristics of automobiles). This creates a classic asymmetric information market failure, with which firms are unable to credibly commit to producing high-quality products in exchange for higher prices. An obvious remedy would be for governments to set up testing facilities and apply labels to certify the validity of the firms' quality claims. This would enable firms to produce high-quality goods at high prices for quality-conscious consumers as well as low-quality products to consumers, who care less about quality.

However, I show in this paper that when the market is imperfectly competitive, a Minimum Quality Standard (MQS), which retains the information asymmetry and therefore forces all consumers to purchase the same homogeneous product, is acutally superior to a label providing full information. What is more, having a standard with asymmetric information is even better than having a standard with full information! The reason is that with asymmetric information consumers consider the products sold by different firms to be homogeneous and competition is relatively intense. In contrast, with full information the firms can differentiate their products by quality and thereby reduce the competitive pressure. As a result, although the label solves the asymmetric information market failure it introduces another, imperfect competition. The MQS only partially adresses the asymmetric information, but it also prevents the firms from exploiting their market power. It is a trade-off between two market failures, asymmetric information and imperfect competition, and in this paper the latter always dominate.

These results have important policy implications. They show how an intuitively appealing policy response to an asymmetric information market failure, to provide information by applying a label, can actually be inferior to less obvious remedies when additional market failures, such as imperfect competition, exist. The conclusions advocate caution when designing policies to ensure that due consideration is given to how the policies could indirectly affect the competitive environment, and perhaps they also help explain why governments sometimes choose to apply a label and in other circumstances impose a MQS to address seemingly similar problems.

The model in this paper follows the vertical product differentiation literature pioneered by Gabszewicz and Thisse (1979), which investigates imperfectly competive markets characterised by products that are differentiated by quality (as opposed to horizontal differentiation, according to which goods are just different, not necessarily better or worse). Shaked and Sutton (1982) showed that vertical differentiation can be used to alleviate price competition and that a natural duopoly can earn positive profits even with free entry, constant marginal costs and arbitrarily small sunk costs. A string of authors have since then expanded on the model. Motta (1993) generalised the model framework to include quality competition (Cournot) as an alternative to price competition (Bertrand) as well as quality-dependent fixed or marginal costs. He found that the results of Shaked and Sutton (1982) are qualitatively robust (albeit less extreme) to the alternative model specifications. Ronnen (1991) included quality-dependent fixed costs and concluded that the market equilibrium produced too much product differentiation (and thus bestowed too much market power on firms) and that a MQS enhances welfare by raising quality and reducing product differentiation. Crampes and Hollander (1995) investigated the use of a MQS with quality-dependent marginal costs and found that in contrast to Ronnen (1991) a MQS may not always benefit consumers. Boom (1995) added the international trade dimension and showed how asymmetric quality regulation in different countries could have spillover effects in otherwise segregated markets.

The main contributions of this paper are to introduce asymmetric information with respect to quality to the vertical differentiation model, and to investigate the welfare impacts of various policy options to address this market failure. My model can be seen as a generalisation of the framework in the sense that it incorporates the specification by Crampes and Hollander (1995) as a special case.

2 Method

My model builds on the vertical product differentiation model by Gabszewicz and Thisse (1979) and Shaked and Sutton (1982). In their model, a Bertrand duopoly sells a product to a continuum of consumers, who earn different levels of income. By differentiating their products by quality, the firms alleviate the price competition and earn positive profits. More specifically, my model resembles the version introduced by Crampes and Hollander (1995), in which the two firms cover the whole market (all consumers choose to buy one of the varieties), marginal costs are positive and increasing in quality, and consumer heterogeneity is based on tastes rather than income.

I depart from the previous work of these authors by introducing asymmetric information with respect to product quality, and by analysing and comparing the impact of different policy instruments intended to address the information imperfections.

Asymmetric information turns the vertically differentiated product into a homogeneous product. Consumers are unable to observe the level of quality (before purchase as well as after consumption) and are therefore also unable to distinguish between the output of the two firms. In the absence of regulation the two firms will supply the lowest possible level of quality, due to the “lemons”-problem identified by Akerlof (1970). The two firms would like to supply a higher level of quality in order to capture part of the consumers’ higher willingness to pay, but since consumers have no means of varifying the firms claims of higher quality, they do not find the claims credible. If they did, the firms would have incentives for supplying the lowest level of quality anyway and reap even greater profits.

The market failure provides justification for regulating the market. I consider three policy options:

1. Label
2. Minimum Quality Standard
3. Minimum Quality Standard combined with a label

The obvious remedy to information imperfections would be to provide information in the form of a government label certifying the claims of the firms. For simplicity, I assume that the government can costlessly observe the level of quality and apply the label, so the government label essentially removes the information assymmetries and reverts the market to the full information setting of Crampes and Hollander (1995). Alternatively, the government can enforce a Minimum Quality Standard (MQS), which Leland (1979) shows improves welfare in an Akerlof (1970) model. In my model, the government can costlessly specify the level of quality supplied by the firms to maximise total welfare. However, the information asymmetry persists and the products are still considered homogeneous. A final policy option is to combine the MQS with a label, which amounts to introducing a standard in the full information setting, which Crampes and Hollander (1995) show is welfare improving over the label alone.

3 Model

Consumers are heterogeneous with respect to their taste for quality. Each consumer is represented by their willingness to pay for quality, θ, which is uniformly distributed over the range $[1, \delta]$, $\delta > 1$, with density $f(\theta) = \frac{1}{\delta - 1}$ (the size of the total consumer base as well as the lower bound of the taste range are normalised to unity). The consumers purchase at most one unit of the differentiated product and choose between the varieties offered by the two firms. In principle, consumers could choose not to purchase anything, but for simplicity I limit the parameter values to ranges that rule out this outcome. I show in the appendix that assuming $1 < \delta < \frac{9}{5}$ is sufficient for this purpose.

At this point, a small remark in relation to Shaked and Sutton (1982) is in order. They show in their model that there is a specific range of consumer heterogeneity, equivalent to $2 < \delta < 4$, within which exactly two firms can earn strictly positive profits. The parameter range considered in this paper, falls short of this “sweet spot”. In Shaked’s and Sutton’s model, a $\delta < 2$ would imply that the low-quality firm would have to price at marginal costs, while the high-quality firm could still generate positive profits. However, they assume that production is costless, whereas I consider marginal costs that are quadratic in quality. Under this assumption, I will show that not only can both firms earn positive profits as long as $\delta > 1$, they earn exactly the same profits.

Consumers' utility is given by

$$U = \theta u_i - p_i \tag{1}$$

where u_i is the quality of the chosen variety, i, and p_i is its price. Each firm supplies one variety at constant marginal costs given by $c_i = u_i^2$. I denote the two varieties by u_h and u_l, with $u_h \geq u_l$, for the high- and low-quality variety, respectively. Both firms are completely identical, so there is no need to identify exactly which firm supplies the high-quality variety and which the low-quality variety. The government chooses between three policy options, the label, the standard, and the combined label and standard, to maximise total welfare given by

$$W = CS + \pi_h + \pi_l \tag{2}$$

where CS is aggregate consumer surplus and π_i is the profits of firm i, $i = h, l$.

The model constitutes a three stage game. In the first stage, the government chooses policy. In the second stage, both firms observe the government policy and choose quality levels in conformity with regulation. Finally, in the third stage the two firms observe the quality levels supplied by the firms and choose prices. In the absence of regulation, the two varieties are identical with quality equal to zero. As a result, the firms play a standard Bertrand game with a homogeneous product, in which prices are set at marginal costs (equal to zero), firms earn zero profits and consumers derive zero utility. I will briefly sketch the equilibrium of the different policy options below.

3.1 Minimum Quality Standard

With a MQS, the two firms sell a homogeneous product priced at marginal costs, $p_h = p_l = u^2$, where u is the quality of the homogeneous product. As the supply curves of the two firms are horizontal and identical, the sales of each firm is unidentified. However, it does not really matter, as profits are zero anyway, so we can basically ignore the two firms. A consumer with taste parameter, θ, earns utility $U(\theta) = \theta u - u^2$ and aggregate consumer surplus (equal to total welfare) becomes

$$\begin{aligned} CS_{MQS} &= \int_1^\delta \left(\theta u - u^2\right) f(\theta)\, d\theta \\ &= \frac{1}{(\delta - 1)} u \left(\frac{1}{2}\delta^2 - \delta u - \frac{1}{2} + u\right) \\ &= \frac{1}{(\delta - 1)} u \left(\frac{1}{2}\left(\delta^2 - 1\right) - u(\delta - 1)\right) \\ &= u\left(\frac{1}{2}(\delta + 1) - u\right) \end{aligned} \tag{3}$$

The first order condition for maximising total welfare is thus

$$\frac{\partial CS_{MQS}}{\partial u} = \frac{1}{2}(\delta+1) - 2u = 0 \tag{4}$$

and the optimal standard is

$$u_{MQS}* = \frac{1}{4}(\delta+1) \tag{5}$$

generating equilibrium total welfare equal to

$$\begin{aligned} CS_{MQS} &= \frac{1}{4}(\delta+1)\left(\frac{1}{4}(\delta+1)\right) \\ &= \frac{1}{16}(\delta+1)^2 \end{aligned} \tag{6}$$

3.2 Label

If the government applies a label, consumers obtain full information about the products and the firms may differentiate their varieties. The firms make their choices during two sequential stages. In the first stage, they determine simultanously the quality of their varieties, and in the second stage, having observed the qualities offered, they set prices (also simultaneously).

The consumers can now observe the quality of the two varieties, and they choose the one that generates the highest utility given their tastes for quality. I denote by $\tilde{\theta}$ the marginal consumer, who is indifferent between consuming any of the two varieties. She is defined as

$$\begin{aligned} \tilde{\theta}u_h - p_h &= \tilde{\theta}u_l - p_l \quad \Leftrightarrow \\ \tilde{\theta} &= \frac{p_h - p_l}{u_h - u_l} \end{aligned} \tag{7}$$

Consumers, whose willingness to pay for quality is larger than that of the marginal consumer, strictly prefer the high-quality variety and therefore chooses this. Conversely, demand for the low-quality variety is generated by all consumers with taste parameter $\theta < \tilde{\theta}$. Aggregate demand for the two varieties is therefore

$$q_h = \int_{\tilde{\theta}}^{\delta} f(\theta)\, d\theta = \frac{1}{\delta-1}\left(\delta - \frac{p_h - p_l}{u_h - u_l}\right) \tag{8}$$

$$q_l = \frac{1}{\delta-1}\left(\frac{p_h - p_l}{u_h - u_l} - 1\right) \tag{9}$$

In the price-setting stage, the firms take quality levels as given and set prices to maximise profits given by

$$\pi_h(p_h, p_l, u_h, u_l) = (p_h - c_h)\frac{1}{\delta-1}\left(\delta - \frac{p_h - p_l}{u_h - u_l}\right) \tag{10}$$

$$\pi_l(p_h, p_l, u_h, u_l) = (p_l - c_l)\frac{1}{\delta-1}\left(\frac{p_h - p_l}{u_h - u_l} - 1\right) \tag{11}$$

I show in the appendix that the Bertrand-Nash equilibrium prices given quality levels can be written as

$$p_h(u_h, u_l) = \frac{1}{3}((u_h - u_l)(2\delta - 1) + 2c_h + c_l) \quad (12)$$

$$p_l(u_h, u_l) = \frac{1}{3}((u_h - u_l)(\delta - 2) + c_h + 2c_l) \quad (13)$$

which provide price-stage equilibrium profit levels of

$$\pi_h(u_h, u_l) = \frac{1}{9(\delta - 1)}(u_h - u_l)[a(2\delta - 1) - u_h - u_l]^2 \quad (14)$$

$$\pi_l(u_h, u_l) = \frac{1}{9(\delta - 1)}(u_h - u_l)[a(\delta - 2) + u_h + u_l]^2 \quad (15)$$

In the quality-setting stage, firms choose quality levels to maximise profits given by (14) and (15). The detailed derivations are presented in the appendix, but it is fairly straightforward to show that the optimal quality levels are given by

$$u_h* = \frac{1}{8}(5\delta - 1) \quad (16)$$

$$u_l* = \frac{1}{8}(5 - \delta) \quad (17)$$

In principle, the exact identities of the high- and the low-quality firms are indeterminate. Either of the firms can become the high-quality firm and be designated firm h, in which case the other firm necessarily has to be firm l. It does not really matter which firm is what as they are completely identical, but in order for (16) and (17) to be Nash equilibrium quality levels, I have to show that no firm has an incentive to unilateral switch place with the other firm, e.g. that firm l has no incentive to leap-frog firm h and become the high-quality producer itself. It turns out that this is particularly easy given the specific functional form chosen for the marginal costs in this model. In the appendix I show that the equilibrium profits are exactly the same for the two firms

$$\pi_h = \pi_l = \frac{3}{16}(\delta - 1)^2 \quad (18)$$

This is not a general result, but follows from the choice of quadratic marginal costs. The high-quality firm charges a higher market price but also faces higher marginal costs. It turns out that these two factors exactly off-set to produce the same profits. Thus, no firm would have any incentives for switching position in the quality spectrum. Crampes and Hollander (1995) show that leap-frogging is also sub-optimal for firms with more general marginal cost functions that are convex in quality.

3.3 MQS combined with a label

Finally, the government may choose to apply a label and, at the same time, impose a MQS. Obviously, if the standard set by the government is lower than quality freely chosen by the low-quality firm, the regulation will be non-binding and the equilibrium resembles the label-only case described above. With a binding MQS, denoted by $\hat{u} > u_l*$ the nature of the strategic interaction changes somewhat. The quality of the low-quality firm is essentially determined by the standard, and the high-quality firm chooses its best-response to the standard, $u_h(\hat{u}) = \frac{1}{3}(2\delta - 1 + \hat{u})$. The stragetic interaction is now between the high-quality firm and the government, with the government acting as a first-mover. Deriving the explicit solution is cumbersome, but it is shown in some detail in the appendix that the equilibrium quality levels are

$$\begin{aligned} \hat{u}* &= \frac{1}{80}\left(65 - 3\sqrt{145} + \left(3\sqrt{145} - 25\right)\delta\right) && (19)\\ &\approx \frac{1}{80}(29 + 11\delta) \\ u_h(\hat{u}*) &= \frac{1}{80}\left(\left(\sqrt{145} + 45\right)\delta - \left(5 + \sqrt{145}\right)\right) && (20)\\ &\approx \frac{1}{80}(57\delta - 17) \end{aligned}$$

4 Comparing the policy options

From the characterisation of the equilibrium outcome of the different policy options investigated above, we see that in all cases, the equilibrium quality levels depend on the degree of dispersion in consumer tastes, δ. Specifically, in the limit as consumers become more homogeneous, $\delta \to 1$, the qualities of both products in the different cases all converge towards the same level, $\tilde{u}* = \frac{1}{2}$. Conversely, when consumers become more heterogeneous in their tastes, the equilibrium qualities under the different policy options diverge. It is instructive to look at the patterns of how the quality levels diverge as δ increases in more detail.

In the case of a label (with no MQS), which is equivalent to a full information setting without regulation, the quality of the high-quality variety increases whereas that of the low-quality variety declines as δ increases. This is the "product differentiation to alleviate price competition"-effect demonstrated by Shaked and Sutton (1982), which is more predominant the higher is δ. With a MQS and no label (i.e. with asymmetric information), product differentiation is not possible, but the optimal standard determining product quality is increasing in δ to accommodate the tastes of the more quality-conscious consumers. Note that the optimal standard is exactly the average quality of what the firms would have chosen with a label and no MQS ($u_{MQS}* = \frac{1}{2}(u_h * + u_l*)$).

Finally, when combining the MQS with a label, the quality of both varieties increase in δ, but the quality of the high-quality variety increases by more than that of the low-quality product. In other words, although product differentiation is possible under the combined policy option, the degree of differentiation is less than when the government only applies the label (no MQS). This is the mechanism generating the positive welfare effects of a MQS inCrampes and

Hollander (1995): starting from a situation with full information (a label), if the government imposes a MQS, it reduces product differentiation and hence the market power of the firms, which in turn increases total welfare.

I am now in a position to present the results of this paper in the form of the following three propositions.

Proposition 1. *a Minimum Quality Standard chosen to maximise total welfare generates more welfare than a label applied to provide full information to consumers.*

Proof. The nature of the proof is straightforward. With specific functional forms for the distribution of consumers and production costs, it is possible to derive explicit solutions for quality levels (as presented above), profits and consumer surplus. Choosing a MQS the government retains the information asymmetry, and the equilibrium reverts to the outcome of a standard Bertrand game with homogeneous goods. Hence, firms earn zero profits and total welfare consists of consumer surplus given by (6)

$$W_{MQS} = \frac{1}{16}(\delta + 1)^2 \tag{21}$$

With a label, consumers face different varieties and consumer surplus becomes more complicated

$$CS_{LBL} = \int_1^{\tilde{\theta}} (\theta u_l - p_l) f(\theta) d\theta + \int_{\tilde{\theta}}^{\delta} (\theta u_h - p_h) f(\theta) d\theta \tag{22}$$

where $\tilde{\theta} = \frac{p_h - p_l}{u_h - u_l}$ is the marginal consumer, who is indifferent between purchasing the high- and the low-quality product. Adding profits of the two firms, I shown in the appendix that total welfare with a label becomes

$$W_{LBL} = \frac{1}{64}\left(\delta^2 + 14\delta + 1\right) \tag{23}$$

What remains is simply to compare total welfare with a MQS and with a label. Define the difference in welfare outcome of the two policy options as

$$\begin{aligned} \Delta_{LBL} &= W_{MQS} - W_{LBL} \\ &= \frac{3}{64}(\delta - 1)^2 > 0 \end{aligned} \tag{24}$$

which is stricly positive for all $\delta > 1$. □

Despite the fact that asymmetric information is clearly reducing total welfare, it is *not* optimal for the government to address the imperfections directly by providing the needed information. It can be explained by reference to a second-best argument. The government struggles with two market failures, asymmetric information and imperfect competition. The information imperfection drives quality levels to a minimum, but it also prevents the firms from exploiting their market power. The label solves one market failure, but introduces another as firms are now able to differentiate their products to alleviate price competition (Shaked and Sutton, 1982). In contrast, the MQS retains the information imperfection with its perfectly competitive outcome, but reduces the adverse impact of the market failure by raising quality.

Crampes and Hollander (1995) show that a MQS can be welfare improving (at the margin) in a full information setting. Translated into my model, they show that a MQS combined with a label is superior to label alone. The obvious question is whether a combined MQS and label is also superior to the MQS alone. The second proposition answers this question.

Proposition 2. *a Minimum Quality Standard chosen to maximise total welfare generates more welfare without a label than when combined with a label.*

Proof. The strategy is the same as in Proposition 1, derive a explicit solutions and compare total welfare under the two policy options. The principle is straightforward, but the manipulations needed are much more cumbersome, so I rely on computer-aided derivation.[1] Define $\Delta_{CH} = W_{MQS} - W_{CH}$ as the difference between welfare under a MQS alone and a MQS combined with a label (as in Crampes and Hollander (1995) - hence the CH subscript). It can be written as

$$\begin{aligned}\Delta_{CH} &= \frac{475 - 29\sqrt{145}}{5760}(\delta - 1)^2 \\ &\approx \frac{127}{5760}(\delta - 1)^2 > 0 \qquad (25)\end{aligned}$$

which is strictly positive for all $\delta > 1$. □

Proposition 2 shows that when the government has the ability to costlessly impose a MQS, asymmetric information is actually welfare improving over full information. The reason is, as above, that full information enables firms to exploit their market power. The distortions arising from imperfect competition outweigh the those from asymmetric information, as the latter are reduced by the MQS.

Proposition 3. *The gains from choosing a MQS without a label over alternative policies increases with the dispersion in consumer tastes, δ.*

Proof. By differentiating (24) and (25) with respect to δ it is easy to see that $\partial\Delta_{LBL}/\partial\delta > 0$ and $\partial\Delta_{CH}/\partial\delta > 0$ for $\delta > 1$. □

Proposition 3 states that the more different are consumers' tastes, the more governments prefer to *avoid* a label, which would allow firms to produce differentiated products. Intuitively, one would expect differentiated products to be preferable when consumers have different taste. However, when firms have market power, greater consumer heterogeneity induces more product differentiation, which in turn increases the distortions arising from imperfect competition.

[1] All derivations are checked using Maple 14 software. An electronic copy or a print of the Maple file can be obtained by contacting the author.

5 Conclusion

This paper demonstrates a few counterintuitive results that arise from the interaction of two market failures, asymmetric information and imperfect competition. Asymmetric information by itself reduces welfare, but it also prevents firms from exploiting their market power. The optimal policy is therefore one that retains the information asymmetry, while reducing the its distorting impact.

In my particular model, the MQS is always the most preferred policy of the three options investigated. In a more general framework the picture may be more nuanced. For instance, firms compete in prices, implying that the information asymmetry has a large impact on market power (essentially generating the perfectly competitive outcome). Motta (1993) shows that the "product differentiation to allviate competition"-effect also exists in models buildling on Cournot quantity competition but that it is less extreme than in Bertrand praice competition models. The results of this paper are therefore likely to generalise to quantity competition, but perhaps only for higher values of the consumer heterogeneity parameter, δ.

To obtain explicit solutions necessary for a discrete comparison of the different policy options, I needed to assume specific functional forms for the distribution of consumers tastes and marginal costs of production. The uniform taste distribution is standard practice in the literature (see e.g. Shaked and Sutton (1982); Ronnen (1991); Crampes and Hollander (1995)). Generalising this assumption is unlikely to qualitatively change the results, but it may limit the welfare superiority of the MQS to the higher ranges of δ. For instance, suppose a relatively large mass of consumers is concentrated at a single peak in the distribution, as opposed to the uniform distribution that features no peak. Intuitively, the two firms would try to capture most of these consumers, which would lead to a narrowing in the gap in quality between the products and hence increase the competitive pressure. This would have the same effect as a decline in δ, which does not change the results qualitatively (unless consumers become completely homogeneous, $\delta = 1$). Similarly, results should also be robust to a more general marginal cost function, as long as it is still convex in quality. As mentioned earlier, the finding that the equilibrium profits of the two firms is exactly the same is attributed to the specific functional form, but this outcome is immaterial to the validity of the main results summarised by the three propositions.

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Appendix

The appendix provides greater detail on the derivations of the results in this paper.

Label

Consider first the price-setting stage. For given levels of quality, the two firms choose prices to maximise profits given by

$$\pi_h(p_h, p_l, u_h, u_l) = (p_h - c_h)\frac{1}{\delta - 1}\left(\delta - \frac{p_h - p_l}{u_h - u_l}\right) \tag{26}$$

$$\pi_l(p_h, p_l, u_h, u_l) = (p_l - c_l)\frac{1}{\delta - 1}\left(\frac{p_h - p_l}{u_h - u_l} - 1\right) \tag{27}$$

The first order conditions are

$$\frac{\partial \pi_h}{\partial p_h} = \frac{1}{\delta - 1}\left(\delta - \frac{p_h - p_l}{u_h - u_l} - (p_h - c_h)\frac{1}{u_h - u_l}\right) = 0 \tag{28}$$

$$\frac{\partial \pi_l}{\partial p_l} = \frac{1}{\delta - 1}\left(\frac{p_h - p_l}{u_h - u_l} - 1 - (p_l - c_l)\frac{1}{u_h - u_l}\right) = 0 \tag{29}$$

which constitute a system of two equations in two unknowns (prices) given quality levels. To solve for prices, rewrite the system in matrix-form as

$$\begin{pmatrix} 2 & -1 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} p_h \\ p_l \end{pmatrix} = \begin{pmatrix} (u_h - u_l)\delta + c_h \\ u_h - u_l - c_l \end{pmatrix} \tag{30}$$

and invert to get

$$\begin{pmatrix} p_h \\ p_l \end{pmatrix} = -\frac{1}{3}\begin{pmatrix} -2 & 1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} (u_h - u_l)\,\delta + c_h \\ u_h - u_l - c_l \end{pmatrix} \tag{31}$$

which can be expanded into

$$p_h\,(u_h, u_l) = \frac{1}{3}\left((u_h - u_l)\,(2\delta - 1) + 2c_h + c_l\right) \tag{32}$$

$$p_l\,(u_h, u_l) = \frac{1}{3}\left((u_h - u_l)\,(\delta - 2) + c_h + 2c_l\right) \tag{33}$$

Inserting the equilibrium prices into the profit expressions and using $c_i = u_i^2$, equilibrium profits can be written as

$$\pi_h = \frac{1}{\delta - 1}\left[\frac{1}{3}\left((u_h - u_l)\,(2\delta - 1) + 2c_h + c_l\right) - c_h\right] \times \left[\delta - \frac{(u_h - u_l)\,(\delta + 1) + c_h - c_l}{3\,(u_h - u_l)}\right] \tag{34}$$

$$\pi_l = \frac{1}{\delta - 1}\left[\frac{1}{3}\left((u_h - u_l)\,(\delta - 2) + c_h + 2c_l\right) - c_l\right] \times \left[\frac{(u_h - u_l)\,(\delta + 1) + c_h - c_l}{3\,(u_h - u_l)} - 1\right] \tag{35}$$

$$\Leftrightarrow$$

$$\pi_h = \frac{1}{3\,(\delta - 1)}\left[(u_h - u_l)\,(2\delta - 1) - c_h + c_l\right] \times \left[\frac{(u_h - u_l)\,(2\delta - 1) - c_h + c_l}{3\,(u_h - u_l)}\right] \tag{36}$$

$$\pi_l = \frac{1}{3\,(\delta - 1)}\left[a\,(u_h - u_l)\,(\delta - 2) + c_h - c_l\right] \times \left[\frac{(u_h - u_l)\,(\delta - 2) + c_h - c_l}{3\,(u_h - u_l)}\right] \tag{37}$$

$$\Leftrightarrow$$

$$\pi_h\,(u_h, u_l) = \frac{\left[(u_h - u_l)\,(2\delta - 1) - c_h + c_l\right]^2}{9\,(\delta - 1)\,(u_h - u_l)} \tag{38}$$

$$\pi_l\,(u_h, u_l) = \frac{\left[(u_h - u_l)\,(\delta - 2) + c_h - c_l\right]^2}{9\,(\delta - 1)\,(u_h - u_l)} \tag{39}$$

$$\Leftrightarrow$$

$$\pi_h\,(u_h, u_l) = \frac{1}{9\,(\delta - 1)}\,(u_h - u_l)\left[(2\delta - 1) - u_h - u_l\right]^2 \tag{40}$$

$$\pi_l\,(u_h, u_l) = \frac{1}{9\,(\delta - 1)}\,(u_h - u_l)\left[(\delta - 2) + u_h + u_l\right]^2 \tag{41}$$

In the quality-setting stage, firms choose quality levels to maximise profits given by (40) and (41). The first order conditions are

$$\begin{aligned}\frac{\partial \pi_h}{\partial u_h} &= \frac{1}{9(\delta-1)}\left[(2\delta-1-u_h-u_l)^2-2(u_h-u_l)(2\delta-1-u_h-u_l)\right]=0\\ \frac{\partial \pi_l}{\partial u_l} &= \frac{1}{9(\delta-1)}\left[2(u_h-u_l)(\delta-2+u_h+u_l)-(\delta-2+u_h+u_l)^2\right]=0\end{aligned}$$

$$\Leftrightarrow$$

$$(2\delta-1-u_h-u_l)(2\delta-1-u_h-u_l-2(u_h-u_l)) = 0 \quad (42)$$

$$(\delta-2+u_h+u_l)(2(u_h-u_l)-\delta+2-u_h-u_l) = 0 \quad (43)$$

Each first order condition has two solutions - one resulting in zero profits. These are clearly not profit maximising. So the profit maximising equilibrium must satisfy

$$2\delta-1-3u_h+u_l = 0 \quad (44)$$

$$u_h-3u_l-\delta+2 = 0 \quad (45)$$

Substituting u_h in (44) yields

$$\begin{aligned}2\delta-1+u_l-3(3u_l+\delta-2) &= 0 \quad \Leftrightarrow\\ u_l &= \frac{1}{8}(5-\delta) \quad (46)\end{aligned}$$

which inserted into (45) yields

$$\begin{aligned}u_h &= \frac{3(5-\delta)+8(\delta-2)}{8}\\ &= \frac{1}{8}(5\delta-1) \quad (47)\end{aligned}$$

To derive equilibrium profits of the two firms, insert the equilibrium quality levels (46) and (47) into profit functions (40) and (41) and get

$$\pi_h = \frac{1}{9(\delta-1)}\frac{(5\delta-1)-(5-\delta)}{8}\left((2\delta-1)-\frac{(5\delta-1)+(5-\delta)}{8}\right)^2 \quad (48)$$

$$\pi_l = \frac{1}{9(\delta-1)}\frac{(5\delta-1)-(5-\delta)}{8}\left((\delta-2)+\frac{(5\delta-1)+(5-\delta)}{8}\right)^2 \quad (49)$$

$$\Leftrightarrow$$

$$\pi_h = \frac{1}{9(\delta-1)}\frac{6(\delta-1)}{8}\left(\frac{12\delta-12}{8}\right)^2 \quad (50)$$

$$\pi_l = \frac{1}{9(\delta-1)}\frac{6(\delta-1)}{8}\left(\frac{12\delta-12}{8}\right)^2 \quad (51)$$

$$\Leftrightarrow$$

$$\pi_h = \pi_l = \frac{3}{16}(\delta-1)^2 \quad (52)$$

MQS combined with a label

The government chooses a MQS, $\hat{u}$, which is only binding if $\hat{u} > u_l*$. I will assume that this is the case, and check if the assumption is indeed satisfied once the optimal MQS is obtained. With a binding MQS, the quality of the low-quality variety is given by the standard and that of the high-quality variety is given as the high-quality firms' best response to the standard, hence

$$u_h(\hat{u}) = \frac{1}{3}(2\delta - 1 + \hat{u}) \quad (53)$$

$$u_l = \hat{u} \quad (54)$$

The government, acting as a first-mover, take these best response functions into account when setting $\hat{u}$. Thus the governments problem is to choose the MQS that maximises total welfare given by

$$\begin{aligned} W(\hat{u}) &= CS(\hat{u}) + \pi_h(\hat{u}) + \pi_l(\hat{u}) \\ &= \int_1^{\tilde{\theta}(\hat{u})} (\theta\hat{u} - p_l(\hat{u})) f(\theta)\, d\theta + \int_{\tilde{\theta}(\hat{u})}^{\delta} (\theta u_h(\hat{u}) - p_h(\hat{u})) f(\theta)\, d\theta \\ &\quad + \frac{1}{9(\delta-1)} (u_h(\hat{u}) - \hat{u}) [2\delta - 1 - u_h(\hat{u}) - \hat{u}]^2 \\ &\quad + \frac{1}{9(\delta-1)} (u_h(\hat{u}) - \hat{u}) [\delta - 2 + u_h(\hat{u}) + \hat{u}]^2 \end{aligned} \quad (55)$$

where $\tilde{\theta}(\hat{u}) = \frac{p_h(\hat{u}) - p_l(\hat{u})}{\hat{u} - u_h(\hat{u})}$ is the marginal consumer, who is indifferent between choosing either of the two varieties.

The simple way to obtain the solution to this maximisation problem is to feed (55) along with equilibrium prices, best response functions and the definition of the marginal consumer into a computer and press "maximise". I have done this in Maple 14 to make sure my derivations are correct. In the following, I will describe the derivations done by hand. To keep it manageable, consider each of the components in turn. Consider first Consumer Surplus

$$\begin{aligned} CS(\hat{u}) &= \int_1^{\tilde{\theta}(\hat{u})} (\theta\hat{u} - p_l(\hat{u})) f(\theta)\, d\theta + \int_{\tilde{\theta}(\hat{u})}^{\delta} (\theta u_h(\hat{u}) - p_h(\hat{u})) f(\theta)\, d\theta \\ &= \frac{1}{\delta-1} \left(\frac{1}{2}\tilde{\theta}(\hat{u})^2 \hat{u} - \tilde{\theta}(\hat{u}) p_l(\hat{u}) - \frac{1}{2}\hat{u} + p_l(\hat{u}) \right) \\ &\quad + \frac{1}{\delta-1} \left(\frac{1}{2}\delta^2 u_h(\hat{u}) - \delta p_h(\hat{u}) - \frac{1}{2}\tilde{\theta}(\hat{u})^2 u_h(\hat{u}) + \tilde{\theta}(\hat{u}) p_h(\hat{u}) \right) \\ &= \frac{1}{\delta-1} \left(\delta \left(\frac{1}{2}\delta u_h(\hat{u}) - p_h(\hat{u}) \right) - \left(\frac{1}{2}\hat{u} - p_l(\hat{u}) \right) \right) \\ &\quad - \frac{1}{\delta-1} \left(\frac{1}{2}\tilde{\theta}(\hat{u})^2 (u_h(\hat{u}) - \hat{u}) - \tilde{\theta}(\hat{u}) (p_h(\hat{u}) - p_l(\hat{u})) \right) \end{aligned} \quad (56)$$

Split the expression for consumer surplus into two blocks, such that $CS(\hat{u}) =$

$A+B$ where

$$A \equiv \frac{1}{\delta-1}\left(\delta\left(\frac{1}{2}\delta u_h(\hat{u}) - p_h(\hat{u})\right) - \left(\frac{1}{2}\hat{u} - p_l(\hat{u})\right)\right) \tag{57}$$

$$B \equiv -\frac{1}{\delta-1}\left(\frac{1}{2}\tilde{\theta}(\hat{u})^2(u_h(\hat{u})-\hat{u}) - \tilde{\theta}(\hat{u})(p_h(\hat{u})-p_l(\hat{u}))\right) \tag{58}$$

The latter block can be written in more manageble terms

$$\begin{aligned} B &= -\frac{1}{\delta-1}\left[\frac{p_h(\hat{u})-p_l(\hat{u})}{u_h(\hat{u})-\hat{u}}\left(\frac{1}{2}(p_h(\hat{u})-p_l(\hat{u})) - (p_h(\hat{u})-p_l(\hat{u}))\right)\right] \\ &= \frac{1}{2}\frac{1}{\delta-1}\frac{(p_h(\hat{u})-p_l(\hat{u}))^2}{u_h(\hat{u})-\hat{u}} \\ &= \frac{1}{2}\frac{1}{\delta-1}\frac{\left((u_h(\hat{u})-\hat{u})(\delta+1)+u_h(\hat{u})^2-\hat{u}^2\right)^2}{9(u_h(\hat{u})-\hat{u})} \\ &= \frac{1}{2}\frac{1}{9(\delta-1)}(u_h(\hat{u})-\hat{u})(\delta+1+u_h(\hat{u})+\hat{u})^2 \end{aligned} \tag{59}$$

Defining another two blocks for the profits of the two firms

$$C \equiv \frac{1}{9(\delta-1)}(u_h(\hat{u})-\hat{u})\left[2\delta-1-u_h(\hat{u})-\hat{u}\right]^2 \tag{60}$$

$$D \equiv \frac{1}{9(\delta-1)}(u_h(\hat{u})-\hat{u})\left[\delta-2+u_h(\hat{u})+\hat{u}\right]^2 \tag{61}$$

it follows that total welfare can be written as the sum of the four blocks, and that the first order condition can be obtained by differentiating each of the four blocks with respect to $\hat{u}$ and setting the sum of the results equal to zero.

Consider first block A

$$\frac{dA}{d\hat{u}} = \frac{1}{\delta-1}\left(\frac{1}{2}\delta^2\frac{du_h(\hat{u})}{d\hat{u}} - \delta\frac{dp_h(\hat{u})}{d\hat{u}} - \frac{1}{2} + \frac{dp_l(\hat{u})}{d\hat{u}}\right) \tag{62}$$

It follows from (53) above that $\frac{du_h(\hat{u})}{d\hat{u}} = \frac{1}{3}$. Prices are slightly more complicated. From (32) and (33), equilibrium prices as functions of the standard can be written as

$$p_h(\hat{u}) = \frac{1}{3}\left((u_h(\hat{u})-\hat{u})(2\delta-1)+2u_h(\hat{u})^2+\hat{u}^2\right) \tag{63}$$

$$p_l(\hat{u}) = \frac{1}{3}\left((u_h(\hat{u})-\hat{u})(\delta-2)+u_h(\hat{u})^2+2\hat{u}^2\right) \tag{64}$$

Differentiating those with respect to $\hat{u}$ yields

$$\begin{aligned} \frac{dp_h(\hat{u})}{d\hat{u}} &= \frac{1}{3}\left[-\frac{2}{3}(2\delta-1)+\frac{4}{9}(2\delta-1+\hat{u})+2\hat{u}\right] \\ &= \frac{1}{27}(22\hat{u}-2(2\delta-1)) \end{aligned} \tag{65}$$

$$\begin{aligned}
\frac{dp_l(\hat{u})}{d\hat{u}} &= \frac{1}{3}\left[-\frac{2}{3}(\delta-2)+\frac{2}{9}(2\delta-1+\hat{u})+4\hat{u}\right] \\
&= \frac{1}{27}(38\hat{u}+2(5-\delta)) \qquad (66)
\end{aligned}$$

Inserting these expressions into (62) yields

$$\begin{aligned}
\frac{dA}{d\hat{u}} &= \frac{1}{\delta-1}\left(\frac{1}{6}\delta^2-\delta\frac{1}{27}(22\hat{u}-2(2\delta-1))-\frac{1}{2}+\frac{1}{27}(38\hat{u}+2(5-\delta))\right) \\
&= \frac{1}{54(\delta-1)}\left(9\delta^2-44\delta\hat{u}+8\delta^2-4\delta-27+76\hat{u}+20-4\delta\right) \\
&= \frac{1}{54(\delta-1)}\left(17\delta^2-8\delta-44\delta\hat{u}+76\hat{u}-7\right) \qquad (67)
\end{aligned}$$

Consider now blocks B, C, and D in turn

$$\begin{aligned}
\frac{dB}{d\hat{u}} &= -\frac{1}{27(\delta-1)}\left(\delta+1+\frac{1}{3}(2\delta-1+\hat{u})+\hat{u}\right)^2 \\
&\quad +\frac{4}{27(\delta-1)}\left(\frac{1}{3}(2\delta-1+\hat{u})-\hat{u}\right)\left(\delta+1+\frac{1}{3}(2\delta-1+\hat{u})+\hat{u}\right) \\
&= \frac{1}{81(\delta-1)}(5\delta+2+4\hat{u})(\delta-2-4\hat{u}) \\
&= \frac{1}{81(\delta-1)}\left(5\delta^2-8\delta-16\delta\hat{u}-16\hat{u}-16\hat{u}^2-4\right) \qquad (68)
\end{aligned}$$

$$\begin{aligned}
\frac{dC}{d\hat{u}} &= -\frac{2}{27(\delta-1)}\left(2\delta-1-\frac{1}{3}(2\delta-1+\hat{u})-\hat{u}\right)^2 \\
&\quad -\frac{8}{27(\delta-1)}\left(\frac{1}{3}(2\delta-1+\hat{u})-\hat{u}\right)\left(2\delta-1-\frac{1}{3}(2\delta-1+\hat{u})-\hat{u}\right) \\
&= -\frac{4}{81(\delta-1)}(4\delta-2-4\hat{u})(2\delta-1-2\hat{u}) \\
&= -\frac{8}{81(\delta-1)}\left(4\delta^2-4\delta-8\delta\hat{u}+4\hat{u}+4\hat{u}^2+1\right) \qquad (69)
\end{aligned}$$

$$\begin{aligned}
\frac{dD}{d\hat{u}} &= -\frac{2}{27(\delta-1)}\left(\delta-2+\frac{1}{3}(2\delta-1+\hat{u})+\hat{u}\right)^2 \\
&\quad +\frac{8}{27(\delta-1)}\left(\frac{1}{3}(2\delta-1+\hat{u})-\hat{u}\right)\left(\delta-2+\frac{1}{3}(2\delta-1+\hat{u})+\hat{u}\right) \\
&= \frac{2}{81(\delta-1)}(5\delta-7+4\hat{u})(\delta+1-4\hat{u}) \\
&= \frac{2}{81(\delta-1)}\left(5\delta^2-2\delta-16\delta\hat{u}+32\hat{u}-16\hat{u}^2-7\right) \qquad (70)
\end{aligned}$$

All that remains now is to sum the four blocks differentiated to obtain the first order condition

$$\begin{aligned}\frac{dW(\hat{u})}{d\hat{u}} &= \frac{dA}{a\hat{u}}+\frac{dB}{d\hat{u}}+\frac{dC}{d\hat{u}}+\frac{dD}{d\hat{u}} \\ &= \frac{1}{54(\delta-1)}\left(17\delta^2-8\delta-44\delta\hat{u}+76\hat{u}-7\right) \\ &\quad+\frac{1}{81(\delta-1)}\left(17\delta^2+20\delta+16\delta\hat{u}+16\hat{u}-80\hat{u}^2-26\right) \\ &= \frac{1}{486(\delta-1)}\left(51\delta^2+48\delta-300\delta\hat{u}+780\hat{u}-480\hat{u}^2-219\right)=0 \qquad (71)\end{aligned}$$

This is, admittedly, not a very pretty expression. It can, however, be shown that the polynomium has two real roots given by

$$\hat{u}_1=\frac{1}{80}\left(65-25\delta+3\sqrt{145}(\delta-1)\right) \qquad (72)$$

$$\hat{u}_2=\frac{1}{80}\left(65-25\delta-3\sqrt{145}(\delta-1)\right) \qquad (73)$$

To investigate further, derive the second order condition for the welfare maximisation problem as

$$\frac{d^2W(\hat{u})}{d\hat{u}^2} = \frac{1}{486(\delta-1)}(780-300\delta-960\hat{u}) \qquad (74)$$

Evaluated at the first root, the second order condition becomes

$$\frac{d^2W(\hat{u}_1)}{d\hat{u}^2}=-\frac{1}{486(\delta-1)}\left(36\sqrt{145}(\delta-1)\right)<0 \qquad (75)$$

for all $\delta > 1$. It is easily shown that the condition evaluated at the second root has the opposite sign. In other words, $\hat{u}_1$ maximises total welfare, whereas $\hat{u}_2$ is the welfare minimising standard. In conclusion, the optimal regulation generates quality levels for the two varieties of

$$\begin{aligned} u_l &= \hat{u}* = \frac{1}{80}\left(65-3\sqrt{145}+\left(3\sqrt{145}-25\right)\delta\right) \qquad (76)\\ &\approx \frac{1}{80}(29+11\delta) \\ u_h &= \frac{1}{3}(2\delta-1+\hat{u}*) \\ &= \frac{1}{80}\left(\left(\sqrt{145}+45\right)\delta-\left(5+\sqrt{145}\right)\right) \qquad (77)\\ &\approx \frac{1}{80}(57\delta-17)\end{aligned}$$

Detailed derivations in the proof of proposition 1

I need to derive an expression for equilibrium welfare in the labelling case and compare that with welfare under a MQS (and no label). It is shown above that equilibrium profits can be written as

$$\pi_h = \pi_l = \frac{3}{16}\left(\delta - 1\right)^2 \quad (78)$$

what remains is deriving an expression for consumer surplus. Earlier in the appendix, I showed that to make the derivations more manageable consumer surplus can be split into two blocks, such that $CS\left(u_h, u_l\right) = A\left(u_h, u_l\right) + B\left(u_h, u_l\right)$, where

$$A\left(u_h, u_l\right) \equiv \frac{1}{\delta - 1}\left(\delta\left(\frac{1}{2}\delta u_h - p_h\left(u_h, u_l\right)\right) - \left(\frac{1}{2}u_l - p_l\left(u_h, u_l\right)\right)\right) \quad (79)$$

$$B\left(u_h, u_l\right) \equiv \frac{1}{2}\frac{1}{9\left(\delta - 1\right)}\left(u_h - u_l\right)\left(\delta + 1 + u_h + u_l\right)^2 \quad (80)$$

Consider first the equilibrium prices, given quality levels $u_h = \frac{1}{8}\left(5\delta - 1\right)$ and $u_l = \frac{1}{8}\left(5 - \delta\right)$

$$p_h\left(u_h, u_l\right) = \frac{1}{3}\left(\left(u_h - u_l\right)\left(2\delta - 1\right) + 2u_h^2 + u_l^2\right) \quad (81)$$

$$p_l\left(u_h, u_l\right) = \frac{1}{3}\left(a\left(u_h - u_l\right)\left(\delta - 2\right) + u_h^2 + 2u_l^2\right) \quad (82)$$

$$\Leftrightarrow$$

$$p_h = \frac{1}{3}\left(\frac{6}{8}\left(\delta - 1\right)\left(2\delta - 1\right) + 2\left(\frac{1}{8}\left(5\delta - 1\right)\right)^2 + \left(\frac{1}{8}\left(5 - \delta\right)\right)^2\right) \quad (83)$$

$$p_l = \frac{1}{3}\left(\frac{6}{8}\left(\delta - 1\right)\left(\delta - 2\right) + \left(\frac{1}{8}\left(5\delta - 1\right)\right)^2 + 2\left(\frac{1}{8}\left(5 - \delta\right)\right)^2\right) \quad (84)$$

$$\Leftrightarrow$$

$$p_h = \frac{1}{192}\left(48\left(2\delta^2 - 3\delta + 1\right) + 51\delta^2 - 30\delta + 27\right) \quad (85)$$

$$p_l = \frac{1}{192}\left(48\left(\delta^2 - 3\delta + 2\right) + 27\delta^2 - 30\delta + 51\right) \quad (86)$$

$$\Leftrightarrow$$

$$p_h = \frac{1}{64}\left(49\delta^2 - 58\delta + 25\right) \quad (87)$$

$$p_l = \frac{1}{64}\left(25\delta^2 - 58\delta + 49\right) \quad (88)$$

Insert the equilibrium prices into block A (79) to get

$$\begin{aligned} A(u_h*, u_l*) &= \frac{1}{\delta-1}\left(\delta\left(\frac{1}{16}\delta(5\delta-1)-\frac{1}{64}\left(49\delta^2-58\delta+25\right)\right)\right) \\ &\quad -\frac{1}{\delta-1}\left(\frac{1}{16}(5-\delta)-\frac{1}{64}\left(25\delta^2-58\delta+49\right)\right) \\ &= \frac{1}{64(\delta-1)}\left(\delta\left(-29\delta^2+54\delta-25\right)-20+4\delta+25\delta^2-58\delta+49\right) \\ &= \frac{1}{64(\delta-1)}\left(-29\delta^3+79\delta^2-79\delta+29\right) \\ &= -\frac{1}{64}\left(29\delta^2-50\delta+29\right) \qquad (89) \end{aligned}$$

Next, insert equilibrium quality levels into block B (80)

$$\begin{aligned} B(u_h*, u_l*) &= \frac{1}{2}\frac{1}{9(\delta-1)}\left(\frac{6}{8}(\delta-1)\right)\left(\delta+1+\frac{4}{8}(\delta+1)\right)^2 \\ &= \frac{6}{64}(\delta+1)^2 \qquad (90) \end{aligned}$$

Finally, adding the two blocks, we can write equilibrium consumer surplus as

$$\begin{aligned} CS(u_h*, u_l*) &= A(u_h*, u_l*) + B(u_h*, u_l*) \\ &= \frac{1}{64}\left(-23\delta^2+62\delta-23\right) \qquad (91) \end{aligned}$$

To obtain total welfare, add the profits of the two firms to consumer surplus to get

$$\begin{aligned} W(u_h*, u_l*) &= CS(u_h*, u_l*) + \pi_h(u_h*, u_l*) + \pi_l(u_h*, u_l*) \\ &= \frac{1}{64}\left(-23\delta^2+62\delta-23+24\left(\delta^2-2\delta+1\right)\right) \\ &= \frac{1}{64}\left(\delta^2+14\delta+1\right) \qquad (92) \end{aligned}$$

Parameter values consistent with assumptions

In this model I assume that both firms exist (there is demand for both varieties) and the market is covered (all consumers choose to buy one of the products). Mathematically, these assumptions can be expressed as

$$1 < \tilde{\theta} < \delta \qquad (93)$$

and

$$\frac{p_l}{u_l} \leq 1 \qquad (94)$$

I will show that $1 < \delta \leq \frac{9}{5}$ satisfies the two conditions under all policy options.

Consider first the MQS (without label). In this case, the two firms produce homogeneous goods and sell them at equal prices. The specific market shares of the two firms are undetermined and the marginal consumer $\tilde{\theta}$ is undefined. The dupoly assumption is therefore consistent wtih any value of δ. Prices are given by marginal costs, and the level of quality is determined by the MQS, so the covered market condition can be written as

$$\begin{aligned} \frac{u_{MQS}^2}{u_{MQS}} &= \frac{1}{4}(\delta+1) \leq 1 \quad \Leftrightarrow \\ \delta &\leq 3 \end{aligned} \tag{95}$$

In the case of a label, the marginal consumer in equilibrium can be characterised by

$$\begin{aligned} \tilde{\theta} &= \frac{p_h - p_l}{u_h - u_l} \\ &= \frac{\frac{1}{3}\left((u_h - u_l)(\delta+1) + u_h^2 - u_l^2\right)}{u_h - u_l} \\ &= \frac{1}{3}(\delta + 1 + u_h + u_l) \\ &= \frac{1}{3}\left(\delta + 1 + \frac{5\delta - 1 + 5 - \delta}{8}\right) \\ &= \frac{1}{3}\left(\delta + 1 + \frac{1}{2}(\delta+1)\right) \\ &= \frac{1}{2}(\delta+1) \end{aligned} \tag{96}$$

The marginal consumer is always the median consumer, so the duopoly condition is always satisfied. To evaluate the covered market condition (94), insert expressions for the equilibrium price (88) and quality level

$$\begin{aligned} \frac{1}{64}\left(25\delta^2 - 58\delta + 49\right) &\leq \frac{1}{8}(5-\delta) \quad \Leftrightarrow \\ 25\delta^2 - 50\delta + 9 &\leq 0 \end{aligned} \tag{97}$$

The roots of the polynomium are $\delta = \left(\frac{1}{5}, \frac{9}{5}\right)$, implying that the condition is satisfied within this range (although $\delta > 1$ by definition).

Finally, investigate the case of a MQS combined with a label. The marginal consumer is given by

$$\begin{aligned} \tilde{\theta} &= \frac{1}{3}\left(\delta + 1 + \frac{1}{80}\left(60 - 4\sqrt{145} + \left(4\sqrt{145} + 20\right)\delta\right)\right) \\ &= \frac{1}{60}\left(35 - \sqrt{145} + \left(\sqrt{145} + 25\right)\delta\right) \\ &\approx \frac{1}{60}(23 + 37\delta) \end{aligned} \tag{98}$$

As $\delta \to 1$, $\tilde{\theta} \to 1$ and $\tilde{\theta} \approx 1.5$ for $\delta = \frac{9}{5}$. The duopoly condition is satisfied for both ends of the parameter range ($1 < \delta \leq \frac{9}{5}$) and since (98) is linear in δ, it must also be satisfied for all values of δ within the range. Deriving the parameter range that satisfy the covered market condition (94) is more complicated, and I have not made the calculations by hand. However, computer-aided derivations show that the condition is satisfied for approximately $-0.2738 < \delta < 2.2738$, which includes the range $1 < \delta \leq \frac{9}{5}$ assumed in this paper.[2]

In conclusion, assuming $1 < \delta \leq \frac{9}{5}$ is sufficient for obtaining an equilibrium in which the market is covered and both firms exist in all the cases considered in this paper.

[2]A copy or a print of the Maple-file can be obtained by contacting the author