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# A Simple Multi-attribute Yardstick Auction Without Prior Scoring

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## **Abstract**

We analyze a simple multi-attribute procurement auction that uses yardstick competition to settle prices. Upon receiving the submitted bids the auctioneer computes the yardstick prices (bids) by a linear weighting of the other participants' bids. The auction simplifies the procurement process by reducing the principal's articulation of preferences to simply choosing the most preferred offer as if it was a market with posted prices.

The auction is not incentive compatible. For some bidders, it may be optimal to bid strategically and manipulate the outcome of the auction. By simulations we show that these opportunities are limited. It is only a small fraction of all bidders that may gain by deviating from submitting true cost bids and this fraction is decreasing as the number of bidders increases. Furthermore, for those that may gain from deviating from telling the truth, the mechanism counteracts strategic bidding in both thin and thick markets. In thin markets deviation have to be large and therefore more risky, in thick markets only small deviations are optimal.

The yardstick auction is not efficient nor optimal relative to a situation where the principal articulates his preferences a priori and uses the efficient second score auction. However, the auction approximates efficiency and the cost of not investing sufficient time and money in articulating a scoring function a priori, is also diminishing as the number of bidders increases. Compared with the efficient second score auction, our numerical results suggest that the yardstick auction generates approximately 1% less social values and 2% less private value with 10 or more bidders.

**Keywords:** Multi-attribute auction, yardstick competition, articulation of preferences, simulation.

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# 1 Introduction

Efficient and flexible procurement systems are often crucial for the success of any organization. As a buyer, organizations obviously want to minimize their spending but attributes such as various types of qualities, delivery time etc. may represent other forms of important objectives as well. Consequently, the development of procurement systems faces an ongoing challenge in designing trading systems that facilitate transparent competition on both price and multiple attributes as well as ensuring sufficient flexibility for operational purposes.

A traditional negotiation process allows full flexibility in the two-sided matching of buyers and sellers, but it is typically ill-structured and opaque. Multi-attribute auctions (scoring auctions), on the other hand, specify *a priori* transparent rules for the procurement "game", but obviously allows for less flexibility<sup>1</sup>.

Che (1993) analyzes the two most common scoring auctions: the *first score* and the *second score auction*.<sup>2</sup> These two auctions have many similarities with the first and second *price* auctions. In fact, Che (1993) proves that the *revenue equivalence theorem* also holds for the first and second score auctions<sup>3</sup>. It is well known that the second score auction is efficient and strategy proof and we therefore use the second score auction as benchmark in our analysis of alternative auction designs.<sup>4</sup>

In practice, though, there is no particular reason to expect that the scoring function is known *a priori* by the buyer (principal). The determination of the scoring function may be complicated for several reasons. For instance, when the principal is a single person and the scoring represents the principal's intra-personal trade-offs, these complications are a central topic of a large literature on Multiple Criteria Decision Making (MCDM), see e.g. Tzeng and Huang (2011). When the principal represents a group of persons (e.g., an organization), the construction of the scoring may involve inter-personal conflicts. The complication of this is no less as reflected by the large literature on Social Choice, see e.g. Arrow (1963), Moulin (1991).<sup>5</sup>

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<sup>1</sup>See e.g. Bichler et al. (2003), Teich et al. (2004) and Teich et al. (2006) for a more elaborated introduction and presentation of several mechanisms that incorporate multiple attributes.

<sup>2</sup>In a first score auction, the bidder with the highest score wins and has to meet the highest score. In a second score auction, the bidder with the highest score wins and has to meet the second highest score.

<sup>3</sup>Using the revenue equivalence theorem as it is presented in Riley and Samuelson (1981).

<sup>4</sup>However, it is not given that the Second Score auction is the most preferred auction by the principal. Bogetoft and Nielsen (2008) show that it is possible for the principal to extract more informational rent while the auction remains efficient and strategy proof.

<sup>5</sup>Empirical cases support the claim that determination of a scoring function is a difficult matter. For instance, in the conservation reserve program the USDA (United States Department of Agriculture) rank the bids into a score. The actual determination of these scoring rules has been widely discussed, see e.g. Babcock et al. (1996, 1997). The applied scoring has also been an issue in the wholesale market for electricity in California. Here the choice of

In this paper we analyze a simple multi-attribute auction (the yardstick auction) that does not rely on well specified preferences (scoring) of the buyer *a priori*. The yardstick auction was introduced in Bogetoft and Nielsen (2003) and it basically replaces the principal's scoring with yardstick competition: Sellers (agents) submit price-bids that are replaced with yardstick prices estimated by a simple linear weighting of the other sellers' bids when applicable. The preprocessed bids (the yardstick bids) are reviewed by the principal. The auction ends by the principal simply selecting the most preferred yardstick bid and the winning seller is compensated with the yardstick price of the winning bid.

However, the yardstick auction mechanism is not strategy-proof. As discussed in Bogetoft and Nielsen (2008) there may be ways for a bidder to manipulate the auction and gain by deviating from truth-telling so truth-telling is not even a Nash equilibrium<sup>6</sup>.

We focus on a two-dimensional model (price and one additional attribute) and show analytically how a bidder may gain by deviating from truth-telling (*ex post*). Next, we set up a simulation framework in order to examine the extent to which it pays off to deviate from submitting the true cost bid. In the simulation we consider two main scenarios, the most optimal deviation from truth-telling for respectively the bidder with the best chance of becoming the winner (the runner-up) and a random bidder. When doing so, we further introduce the possibility of sellers being technically inefficient in producing the attribute. This aspect matters here since the yardstick prices are determined by convex envelopment of the data points and in practice it is well known that technical inefficiency often occurs.

We demonstrate that although the auction allows room for manipulation, this room is somewhat limited. It is only a small fraction of all bidders that may gain by deviating from truthful bidding and this fraction is decreasing as the number of bidders increases. Furthermore, with as few as 3 bidders those that may benefit from deviating have to deviate a lot. It appears that the mechanism counteracts deviation in both thin and thick markets; in thin markets successful deviations have to be large and therefore more risky from an *ex ante* perspective; in thick markets successful deviations can be smaller but have to be confined within a narrower range again making deviation risky from an *ex ante* perspective.

Moreover, we demonstrate that the average loss from not articulating preferences (i.e. constructing a scoring function) is relatively small both in terms of social and private values. In a thin market with as few as 3 bidders it ranks from 10 to 16% relative to a traditional second score auction with linear scoring. With just 7 bidders the cost decreases to 2-5% and continues to drop as the number of bidders increases.

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an unsuitable scoring rule had severe consequences, see Bushnell and Oren (1994) and Chao (2002).

<sup>6</sup>A third paper using the underlying yardstick principle as part of an auction mechanism is Aparicio et al. (2008).

Therefore we conclude that in a situation where the cost of articulating preferences *a priori* is relative high compared to the social and private values at stake (e.g. when small firms or consumers engage in multi-attribute competition as if it was a market with posted prices), the suggested mechanism seems to represent an attractive alternative in practice to more informationally demanding mechanisms.

The outline of the paper is as follows. Section 2 introduces the procurement setting and the yardstick prices. Section 3 describes the yardstick auction and shows analytically then deviating from truth-telling is optimal (ex post). Section 4 introduces the simulation framework and the results are presented in Section 5, while Section 6 concludes.

## 2 Setting

In this section, the procurement setting and the applied yardstick prices are presented.

### 2.1 Procurement setting

We consider a risk neutral principal seeking to procure a good or a project from one of  $n$  risk neutral agents,  $i \in N = \{1, \dots, n\}$ .

In particular, we consider the simplified case where the properties of the good offered by agent  $i$  are given by a one-dimensional variable  $y^i \in \mathbf{R}_0$ . We may for example consider  $y^i$  as the quality level of the (fixed amount of) output delivered.

To focus on the adverse selection problems we further assume that delivery of the promised qualities can be costlessly enforced (e.g. by a harsh penalty for deviations).

To produce the output level  $y^i$ , agent  $i$  needs inputs. Inputs will be aggregated into a single dimensional cost  $c^i \in \mathbf{R}_0$ . The production cost  $c^i$  is private information to agent  $i$ . All involved parties know that  $c = (c^1, \dots, c^n)$  belongs to some unknown common cost structure  $\mathbb{C} \subseteq \mathbf{R}_0^n$ .

Since the actual quality level  $y^i$  is verifiable, possible manipulations regard the costs. Therefore, the signal from agent  $i$  is simply a price-quality bid

$$(x^i, y^i) \in \mathbf{R}_0^2$$

with the interpretation that agent  $i$  is willing to produce quality level  $y^i$  if he is paid at least  $x^i$ .

We assume throughout that the aim of the agent is to maximize (expected) profit:

$$\pi^i = b^i - c^i$$

where  $b^i$  is the payment to bidder  $i$ .

The aim of the principal is to maximize (expected) net value, i.e., the value generated by the good minus the costs of acquiring it. If the principal is a social planner seeking to maximize social value, the objective value when agent  $i$  is selected is:

$$V(y^i) - c^i$$

where  $V(\cdot)$  is a weakly increasing value function. More generally, we could let the objective be  $V(y^i) - (1 + \epsilon)c^i$ , where  $\epsilon \geq 0$  reflects the possible costs of distortions from the collection of public funds. Since it has no influence on the results—except that all costs are increased by a factor  $(1 + \epsilon)$ —we simply assume that  $\epsilon = 0$ .

If the principal is a private, profit maximizing entity, the net value when agent  $i$  is selected, is:

$$V(y^i) - b^i$$

i.e. the value minus the payment to agent  $i$ .

## 2.2 The Bidders' Cost Structure and Yardstick Prices

As mentioned above, it is common knowledge that the reported costs of the agent (as exposed in the bid  $x^i$ ) comes from some underlying common cost structure represented by a cost function  $C(y)$ .

We assume that  $C(y) = \min\{x \mid (x, y) \text{ is feasible}\}$  is:

- A1. Weakly increasing :  $y' \geq y \Rightarrow C(y') \geq C(y)$ ,
- A2. Convex :  $C(\gamma y + (1 - \gamma)y') \leq \gamma C(y) + (1 - \gamma)C(y'), \forall \gamma \in [0, 1]$ .

A number of different cost models satisfy these two simple requirements, but we will assume that a trusted third party (e.g. the auctioneer) estimates the cost function using a convex envelopment of the observed bids (that is the smallest of such models including all observed data) as illustrated in Figure 1.

For each agent  $i \in N$  a yardstick price  $C^{-i}(y^i)$ , as illustrated in Figure 2, is defined as follows:

**Definition 1.** For all  $i \in N$  the yardstick price  $\bar{x}^i$  is computed as:

$$\bar{x}^i = C^{-i}(y^i) = \inf\{x \in \mathbf{R}_0 \mid x \geq \sum_{j \in I, j \neq i} \lambda^j x^j, \quad y^i \leq \sum_{j \in I, j \neq i} \lambda^j y^j, \quad \sum \lambda = 1, \lambda^j \geq 0\}. \quad (1)$$

The above solution identifies a single point  $(\bar{x}^i, y^i)$  on the spanned frontier (being the smallest convex envelopment of all bids except for bidder  $i$ 's bid).

Note that for some values of  $y$ , the associated yardstick costs may be infinite (e.g., bid  $E$  in the figure). In the yardstick auction presented in Section 3 this is dealt with by asking the principal to announce an upper bound on the bids, i.e. the highest value of  $y$  and its associated reservation price  $x$  that the principal is willing to accept.

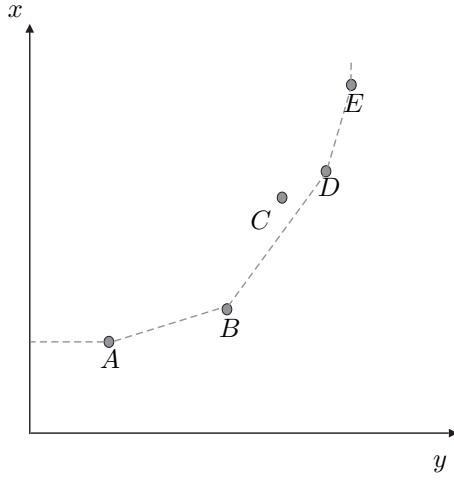


Figure 1: *Convex cost structure*

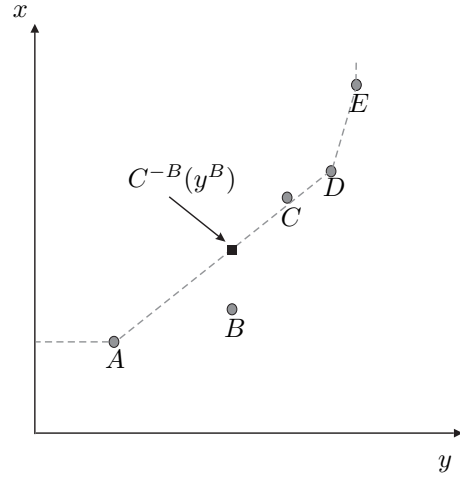


Figure 2: *Yardstick cost*

### 3 The Yardstick Auction

In this section, we present the rules defining the yardstick auction and show analytically how bidders may gain by deviating from truth-telling in this auction.

#### 3.1 The Yardstick Auction:

The rules defining the auction are provided by the stepwise procedure below.

**Step 0:** The principal announces the procurement proposal and an upper bound on the bids  $z^P = (y^P, x^P)$ , where  $y^P$  is the highest acceptable value of  $y$  and  $x^P$  is the highest acceptable price for  $y^P$ .

**Step 1:** Each participant  $i \in N$  submit a single sealed bid  $z^i = (x^i, y^i)$  to a Trusted Third Party (TTP). Let  $Z$  be the set of bids including  $z^P$ , i.e.,  $Z = \{z^i\}_{i \in N} \cup z^P$ .

**Step 2:** The TTP computes a yardstick price  $\bar{x}^i = C^{-i}(y^i)$  for all  $i \in N$  (see definition 1) that replaces  $x^i$  when  $x^i \leq \bar{x}^i$ . Let  $\bar{z}^i = (\bar{x}^i, y^i)$  be the yardstick bid of agent  $i \in N$  and let  $\bar{Z}$  be the set of such yardstick bids.

**Step 3:** The set  $\bar{Z}$  is presented to the principal, who selects the winning bid  $\bar{z}^{i^{**}}$ .

**Step 4:** The winner  $i^{**}$  is compensated by  $\bar{x}^{i^{**}}$  and losers are not compensated.



In Step 0, the principal starts the auction by publicly announcing  $z^P$  stating the maximum value of the attribute in question  $y^P$  and its reservation price  $x^P$  for  $y^P$ .  $z^P$  enters the auction both as a submitted bid as well as a yardstick bid. Thereby,  $x^P$  addresses the problem of non-existing yardstick price for the maximal value of  $y$  among the bidders.

The remaining parts of the auction run as follows: In Step 1, the bidders submit their sealed bids. In Step 2, the yardstick prices ( $C^{-i}(y^i)$ ) are computed (as illustrated in Figure 2). If  $C^{-i}(y^i) \geq x^i$  the bidders are assigned a yardstick bid by replacing the submitted price-bid with the computed yardstick price. In Step 3, the principal reviews the yardstick bids and selects a single yardstick bid as the winner of the auction. Step 4, finalizes the auction by compensating the selected winner with his yardstick price.

The auction may be seen as a mechanism for settling posted prices on goods and services with linear weighting of price and other attributes. In fact in comparing with the second score auction, the mechanism settles the most pessimistic prices seen from the principal's point of view. To see this, note that the yardstick prices are equal to the the highest possible second score compensation, i.e. the scoring function that is just spanned by the other agents' bids in the exact same ways as the common cost structure is estimated in Definition 1.

Therefore, due to the use of linear scoring functions, there will always be a cost of *not* articulating a scoring function a priori (we measure this in Section 5).

### 3.2 Strategic Behavior

We now turn towards issues related to bidders' possible strategic behavior. As mentioned in the introduction, we consider the situation where it is impossible (or costly) for the principal to articulate a scoring function *a priori*. However, we assume that it is possible for the principal to make a unique selection in Step 3 of the mechanism, which is consistent with some kind of underlying concave scoring function.

Suppose that the agents have submitted their true bids  $z^i = (x^i, y^i)$  and yardstick bids  $\bar{z}^i = (\bar{x}^i, y^i)$  have been determined adding the point  $z^P$  to the observed data. As shown below truth-telling is not a Nash equilibrium.

**Proposition 1:** *Consider a given agent  $i \in N$ . By increasing the price bid  $x^i$  up to at most the yardstick price  $\bar{x}^i$  both neighbor yardstick bids increase and thereby weakly increases  $i$ 's change of winning the auction.*

*Proof:* Consider an arbitrary agent  $i \in N$ . First, note that changes in  $i$ 's price bid  $x^i$  has no influence on  $i$ 's computed yardstick bid  $\bar{z}^i$  (as this is determined excluding agent  $i$ 's bid from the data set).

Now, if  $i$  increases his price bid to  $x'^i > x^i$  ( $x'^i \leq \bar{x}^i$ ), then both  $i$ 's neighbors will obtain higher yardstick price bids: Indeed,  $i$  has a neighbor yardstick bid to the left  $\bar{z}^{i^l}$  and to the right  $\bar{z}^{i^r}$  (these always exist since  $z^P$  and  $z^0$  are added to the observed data). The yardstick bid,  $\bar{z}^{i^l}$ , is determined by  $z^i$  and  $z^{i^{l^l}}$  (or  $z^i$  alone if  $z^{i^{l^l}} = z^0$ ). If  $z^{i^{l^l}} = z^0$  it is clear that  $\bar{x}^{i^l}$  increases when  $x^i$  increases (with the same amount). If  $z^{i^{l^l}} \neq z^0$  we note that the facet spanned by  $z^{i^{l^l}}$  and  $z^i$  becomes steeper as  $z^{i^{l^l}}$  is fixed while  $i$  increases the price bid  $x'^i > x^i$ . Hence,  $\bar{x}^{i^l}$  increases. Now, assume that  $i$  has a neighbor yardstick bid to the right  $\bar{z}^{i^r}$  and let  $z^{i^{r^r}}$  be its neighbor bid to the right (possibly  $z^P$ ). The bid  $z^{i^r}$  is determined by  $z^i$  and  $z^{i^{r^r}}$ . Since the facet spanned by  $z^i$  and  $z^{i^{r^r}}$  becomes flatter as  $i$  increases his price bid while  $z^{i^{r^r}}$  remains fixed, we get that  $\bar{x}^{i^r}$  increases.

Since the yardstick bid of both neighbors increase if  $i$  increases his price bid, both neighbor agents will get non-increasing scores while the other agents (including  $i$ ) have unchanged score. Thus,  $i$ 's chance of winning the auction weakly increases.  $\triangle$

Note that if agent  $i$  bids above his yardstick price bid he will loose the auction for sure.

In a similar fashion we can show that if agent  $i$  decreases his price bid both neighbor agents will get decreasing yardstick price bids which in turn weakly decreases  $i$  chance of winning the auction. Thus, bidding below ones true bid is disadvantageous *unless* the bid is so low that it in fact excludes the neighbor agent from the auction (in the sense that it makes the neighbor agent  $j$ 's yardstick bid go below  $j$ 's true cost). Such a situation is illustrated in figure 3, where  $i$  decreases his bid  $z^i$  to  $z'^i$  and hereby exclude  $j$  from the auction.

We summarize in the following observation.

**Observation 1:** *Since the yardstick price remains unchanged, bidding sufficiently below true costs is therefore always a beneficial deviation from truth-telling unless there are limits to how low bids the principal accepts.*

## 4 The Simulation Framework

In this section, we present the simulation framework that allows us to analyze to what extend it pays off to deviate from truth-telling in the yardstick auction.

First, we randomly generate agents bids. Specifically, agent  $i$ 's attribute value  $y_i$  is independently drawn from the uniform distribution  $\mathcal{U}[1, 10]$  and the agents' cost is determined by the common underlying cost function  $x(y) = Cy^2$ , where  $C$  is measuring the degree of technical inefficiency in production: Initially, we

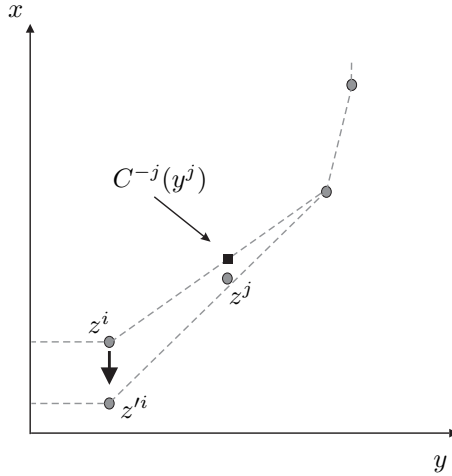


Figure 3: *Excluding the neighbor yardstick bid*

assume no technical inefficiency ( $C = 1$ ), and then proceeds to examine two different levels of technical inefficiency with  $C$  independently drawn from  $\mathcal{U}(1, 1.1)$  and  $\mathcal{U}(1, 1.2)$ , i.e., all bidders initiate from the same common underlying technology with up to 0, 10 and 20% technical inefficiency being introduced via the cost function.

Second, we introduce the principal's value function. We assume that the principal selects a winner of the auction as if there was an underlying linear scoring function given as  $V(y) = 10y$ , where the slope equal to 10 is the slope of the common cost structure for  $y = 5$ .

We simulate the mechanism  $10^3$  times with the number of participants ranging in  $\{3, 4, 5, 7, 10, 15, 20, 30\}$ . For every iteration we simulate the participating agents' bids  $(y, x)$  by randomly drawing  $y$  and  $C$  (depending on the scenario), compute the yardstick bids and corresponding scores, identify the winner of the auction and finally calculate its utility.

Based on the above setting we perform two sets of simulations. In the first set we simulate the misreporting of the runner-up agent's cost (the agent with the second highest score during the auction) and calculate the effect of that misreporting on the agent's utility. In the second set we follow an identical process but for a randomly selected agent instead (it can also be one of the inefficient agents). The range of misreporting varies from 40% to 140% of a selected agent's true cost and increases in an interval of 1%.

To this end, and since a successful manipulation can be both above and below the true cost, we harvest the following five quantities:

**A1 - Successful above:** The percentage of all simulated auctions where the selected agent may win with a gain by bidding up till 140% of true cost.

- A2 - Above to succeed:** The smallest percentage increase of true cost for those who successfully may win with a gain by bidding up till 140% of true cost.
- A3 - Successful Interval:** The interval in percentage points where those who successfully may win with a gain by bidding up till 140% of true cost remains the winner.
- B1- Successful below:** The percentage of all simulated auctions where the selected agent may win with a gain by bidding down to 40% of true cost.
- B2 - Below to succeed:** The smallest percentage decrease of true cost for those who successfully may win with a gain by bidding down to 40% of true cost.

In addition to this, we examine the simulated auctions with respect to efficiency (if the social value is maximized) and the costs to the principal of not stating a scoring function a priori. This we simply do by comparing the outcome of the yardstick auction with that of a second score auction. We use the second score auction as benchmark since it relies on a well-defined scoring function, is efficient and strategy proof, see e.g. Che (1993). The results of this analysis are summarized through the following three quantities:

- E1 - Degree of efficiency:** The percentage of the iterations of the algorithm where the yardstick auction selects the same winner as the second score auction i.e. selecting the bidder that maximizes the social value  $V(y^i) - c^i$ , for more see Section 2.
- E2 - Efficiency cost:** The relative cost in social values, by not being efficient. That is:  $\frac{V(y^{i*}) - c^{i*} - (V(y^{i**}) - c^{i**})}{V(y^{i*}) - c^{i*}}$  where  $i^*$  and  $i^{**}$  is respectively the winner of second score auction and the yardstick auction.
- E3 - Cost of no scoring:** The relative cost to the principal of not constructing a scoring function a priori and then use the second score auction. That is:  $\frac{V(y^{i*}) - b_{SSA}^{i*} - (V(y^{i**}) - b_{YA}^{i**})}{V(y^{i*}) - b_{SSA}^{i*}}$  where  $b_{SSA}^{i*}$  and  $b_{YA}^{i**}$  is respectively the compensation to the winners of the second score auction ( $i^*$ ) and the yardstick auction ( $i^{**}$ ). Hereby we assume that the principal could have constructed (by sufficient investments) the same scoring function that he (indirectly) end up using in the yardstick auction.

Technically, all simulations are done in R and all DEA programs are solved using the "Benchmarking" package for R, cf. Bogetoft and Otto (2011) and Bogetoft and Otto (2012).

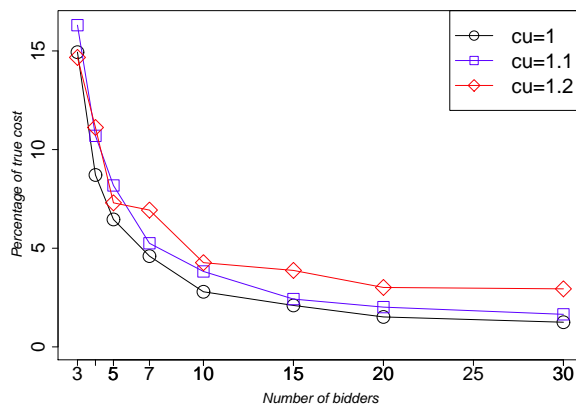


Figure 4: Average deviation above true cost to gain for runner-up

## 5 The Simulation Results

Having described the simulation's input parameters and objectives we now present our numerical findings for the two sets of simulations.

### 5.1 The runner-up

In this Section we consider the bidder with the best possibilities for manipulating the auction (the runner-up), hereby we measure the worst case scenario *ex post*.

Analytically, we know that it may pay off to bid above true cost for a runner-up by making the neighbor bidders look less attractive to the principal. From the simulations we see that with 3 bidders approximately 15% of all runner-ups may gain by bidding above true cost; however they would have to bid more than 14% above on average, and no more than 20% not to exclude themselves (see Figures 4 and 5). As the number of bidders increases, the runner-up may win the auction by a smaller increase but it also becomes more difficult to avoid excluding themselves. For instance, for 10 bidders the runner-up should approximately bid between 4 and 8% above true cost on average (see Figures 4 and 5)<sup>7</sup>. Therefore, bidding above in a thin market requires a large deviation from truth-telling and as the number of participants increases the interval in which deviation is optimal narrows down considerably.

Bidding below true cost is of less concern for the principal who, in our model, only wants to minimize expenditures (provided the seller is able to deliver the

<sup>7</sup>In Figure 5, the interval in which the runner-up may gain is smaller for the for 3 than for 4 bidders and then it decreases as the number of bidders increases. The reason for this comes from the way the mechanism settle prices for the one with the lowest  $y$ .

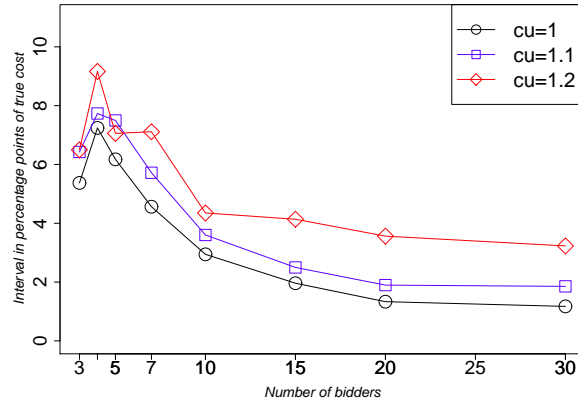


Figure 5: Average interval in percentage points of true cost with gain for runner-up

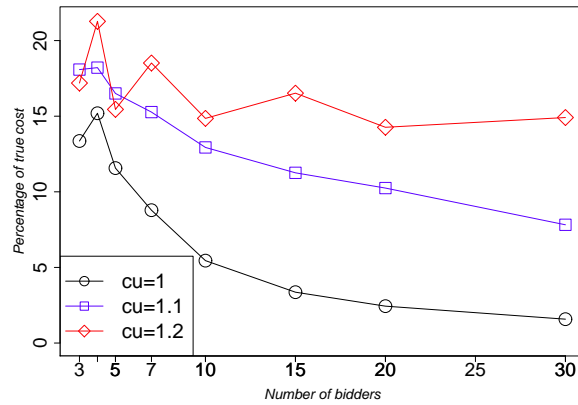


Figure 6: Average deviation below true cost to gain for runner-up

attribute level as promised). Since the runner-up does not influence his own yardstick price and the winner is a neighbor, it always pays off for a runner-up to bid below true cost. However, with 3 bidders, a runner-up has to bid more than approximately 15% below in order to gain (see Figure 6). As the number of bidders increases, it becomes easier for a runner-up to gain by bidding below. Unlike results on bidding above, introducing inefficiency in production has a much higher impact here. The more technical inefficiency, the more it takes to deviate below in order to gain for the runner-up.

## 5.2 A random bidder

As for the runner-up, we also measure the most optimal behavior for a random bidder *ex post*. There is of course a  $1/n\%$  probability of that agent being the runner-up, as there is an equal probability of the selected agent being the actual winner of the auction. Also, a random bidder may be technically inefficient, i.e. a bidder that will have a loss if winning the auction by misreporting (e.g., say the random bidder is the only technically inefficient bidder with a true cost above his yardstick cost, then bidding below he may risk to win the auction being compensated with his yardstick cost).

Unlike the runner-up only around 5% of randomly chosen bidders in the case of 3 bidders would gain by bidding above and this number decreases to less than 1% as the number of bidders increase.

For those that may gain by bidding above their true costs, the average deviation is similar to the results for the runner-up. The results have the same tendency: it is just approximately 0.5% less than the results in Figure 4 and 5. The reason for this is that the main part of those random bidders that may gain by bidding above are in fact the runner-ups.

The effects of under-reporting by a randomly selected agent differs significantly from that of the runner-up though. The percentage that may gain by bidding below is much smaller and the level of technical inefficiency now has a large impact. If all bidders are efficient, bidding below is essentially a "free lunch" as for the runner-up (however in the simulation we only allow the bidders to bid 40% of true cost). But if we introduce technical inefficiency, winning by bidding below may result in a loss. This has a great impact on the percentage that may gain by bidding below. With 3 bidders approximately 30% may gain which decreases to approximately 10% as the number of bidders increases. Finally, for those that gain by bidding below, deviation requires more aggressive bidding as the number of bidders increases, see Figure 7.

## 5.3 Efficiency and cost of no scoring

We now turn to the overall performance of the yardstick auction with respect to efficiency (i.e. maximizing social value) and the costs to the principal from not articulating a scoring function a priori. This we do by presenting measures introduced in Section 4 that compare the outcome of the yardstick auction with that of the second score auction.

First we consider the efficiency of the yardstick auction assuming that the underlying scoring function used for the simulation is in fact the principal's true scoring function.

Initially we measure to what extent the two auctions select the same winner (E1). In the case where some bidders are technically inefficient ( $C = 1.1$  or  $1.2$ ), the percentage of auctions having the same winner increases only a little

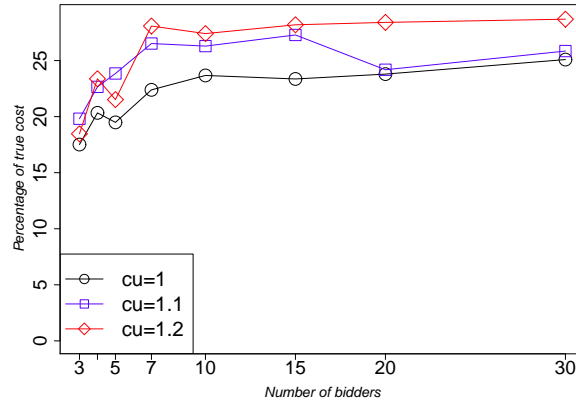


Figure 7: Average deviation below true cost to gain for a random bidder

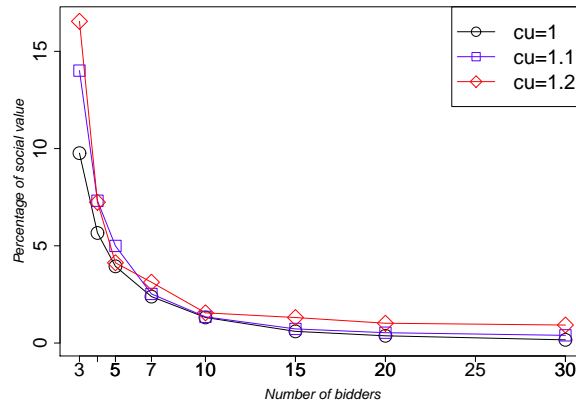


Figure 8: Decrease in social values by using the yardstick auction

with the number of bidders. With only 3 participants 50% of all auctions have the same winner which increases to approximately 70% with 30 bidders. In the special case where all bidders are fully efficient in production ( $C = 1$ ), half of the auctions have the same winner independent of the number of bidders.

Although, the yardstick auction may not select the most efficient winner, we have that the yardstick auction becomes increasingly efficient as the number of bidders increases. This we measure by E2, introduced in Section 4, which captures the average relative loss in social values by using the yardstick auction. Figure 8 shows that E2 decreases from 7-9% with 3 bidders to approximately 1% or less for 10 or more bidders. With four or more bidders this result is independent of whether the bidders are technically efficient or not.



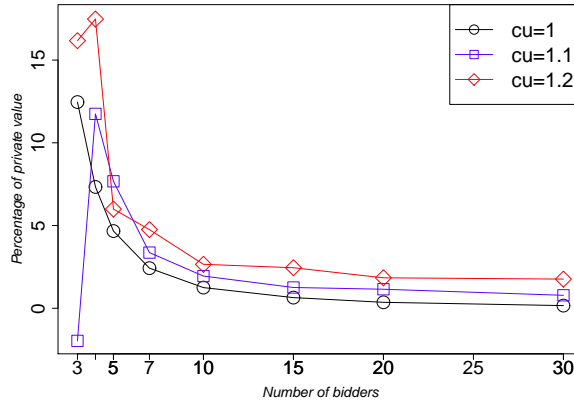


Figure 9: *Decrease in private values by no scoring*

Suppose the principal could have constructed a scoring function *a priori* and used the second score auction instead. The loss by not doing so and using our suggested yardstick auction is measured by E3, introduced in Section 4, which captures the average relative loss in private value of the principal by using the yardstick auction. Figure 9 shows that E3 decreases from 10-14% with 3 bidders to approximately 2% or less for 10 or more bidders. The results are only slightly affected by whether the bidders are technically efficient or not, with a small tendency towards a higher cost the more inefficient the bidders are in production.

As presented in Section 4, the mechanism can be seen as a mechanism that post the most pessimistic prices seen from the principal's point of view given linear scoring. Therefore the E3 measure will always be positive<sup>8</sup>.

## 6 Conclusion

We have analyzed a multi-attribute procurement auction that applies yardstick competition to simplify the principal's weighting of costs and benefits. The auction reduces the cost of articulating preferences to a mere problem of picking a favored alternative (as with posted prices). We measure the possibilities for a bidder to manipulate the auction as well as the overall social and private costs of leaving out scoring.

The average loss from not articulating preferences (i.e. constructing a scoring function) is relatively small both in terms of social and private values. In a thin market with as few as 3 bidders it ranks from 10 to 16% relative to the

<sup>8</sup>Note that this does not hold for strictly concave scoring, which may settle second-score-prices that are higher than the yardstick prices.

traditional second score auction with a linear scoring function. With just 7 bidders the cost decreases to 2-5% and continues to drop as the number of bidders increases.

Moreover, although in theory it may be optimal for the bidders to bid strategically and manipulate the outcome of the auction, the simulations show that these opportunities seem limited in practice. It is only a small fraction of all bidders that may gain by deviating from submitting a true cost bid (bidding above) and this fraction is decreasing as the number bidders increases. Furthermore, the range for which it is optimal to deviate (both above and below) also narrows down rather fast as the number of bidders increases. Hence, from an *ex ante* perspective, it appears that bidders should be careful deviating from the true cost as it easily results in either "winners curse" (if bidding too aggressively below) or self exclusion from the auction (if bidding too aggressively above).

The yardstick auction is not efficient nor optimal relative to a situation where the principal articulates his preferences a priori and uses the efficient second score auction. However, the auction approximates efficiency and the cost of not investing sufficient time and effort in articulating a scoring function *a priori* is also diminishing as the number of bidders increases. With 10 bidders the cost of not announcing a scoring function is less than 2% of the private value to the principal.

Therefore we conclude that in a situation where the cost of articulating preferences *a priori* is relatively large compared to the social and private values involved, the suggested yardstick auction seems to perform rather well in practice and is highly operational compared to other forms of auction designs.

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